chapter

"1"

* Function and Models :-

Def of Function:-

$f : A \to B \Rightarrow$ f is a function from a set A to set B

(f) assigns to each element (عنصر) in A a unique element

* كل عنصر في المجال يرتبط بصورة واحدة فقط في المدى .

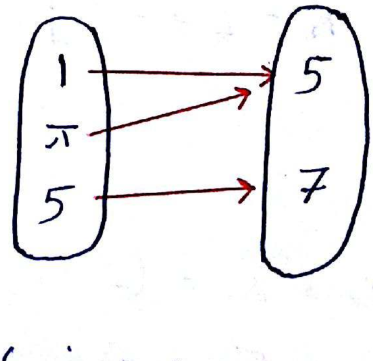
* The domain of f = D_f (المجال)
* The Range of f = R_f (المدى)

$\mathbb{R}$ = set of real number $(-\infty, \infty)$
الاعداد الحقيقية

Ex: Determine whether or not (R) is function, If it is state that domain and Range of it ?

$$R = \{(1,5), (\pi,5), (5,7)\}$$

Sol:



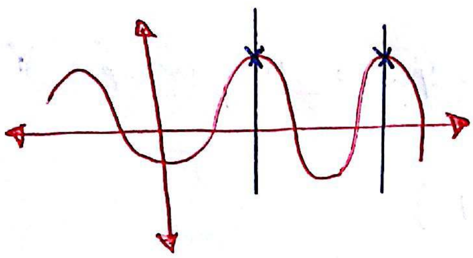
* Domain $\{1, \pi, 5\}$
* Range $\{5, 7\}$

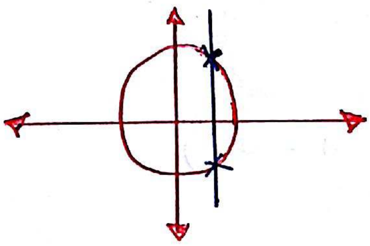
كل عنصر في المجال له صورة واحدة فقط في المدى

Vertical line test :- if the vertical line cuts the curve in only one point, then it's a function other wise it's not function.

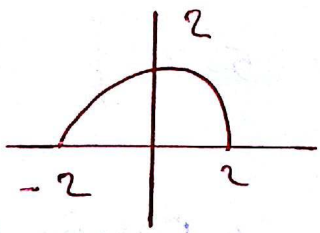
* إختبار الخط العمودي :- اذا قطع الخط الرسمة في أكثر من نقطة فلا يكون اقتران

EX :- which of the following is a function ?

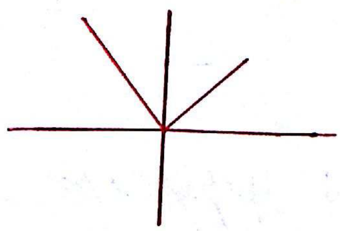
[1]  $\Longrightarrow$ Yes, function

[2]  $\Longrightarrow$ NOT function
(لان قطع بأكثر من نقطة)

EX :- find the domain and the range :-

[1] 

$D = [-2, 2]$
$R = [0, 2]$

[2] 

$D = \mathbb{R} = (-\infty, \infty)$
$R = [0, \infty)$

* عند (∞) دائماً مفتوحة .

$\boxed{*}$ $f(x) = \sqrt{2x-1}$, find the domain ?

$2x - 1 \geq 0$

$2x \geq 1$

$x \geq \frac{1}{2}$

$D = [\frac{1}{2}, \infty)$

* للجذور ذات الأعداد الزوجية

[(ماداخل الجذر) ≥ 0 ماداخل الجذر]

اكبر او يساوي

$\boxed{*}$ $f(x) = \dfrac{2x+7}{x^2 - 3x + 2}$

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

$D = \mathbb{R} - \{1, 2\}$

اصفار المقام

$f(x) = \dfrac{\text{Poly}}{\text{Poly}}$

$D = \mathbb{R} -$ (اصفار المقام)

$\boxed{*}$ $f(x) = \dfrac{3}{\sqrt{x^2 + 5x + 6}}$

$x^2 + 5x + 6 > 0$

$(x+2)(x+3) > 0$

* الجذر بالمقام !

اذن لا نضع اشارة المساواة :)

+++ ---- +++

-3 -2

$D_f = (-\infty, -3) , (-2, \infty)$

or $D_f = \mathbb{R} - [-3, -2]$

$\boxed{*}$ $f(x) = \dfrac{x^2 + 2x}{\sqrt[3]{1 + x^5}}$

إنتبه !

* ~~الجذر قوة (عددية)~~
* الجذر (فردي) : يمكن ان يكون داخله اي عدد موجب او سالب

لأنه في الجذور الزوجية فقط موجب

$x \in \mathbb{R} : x \neq -1$

$D = \mathbb{R} - \{-1\}$

مثال :

$\sqrt[3]{-1} = -1$

$\sqrt[2]{-1} =$ غير معرف

<u>EX</u>: find the equation of line whose x-intersection is (2) and y-intersection is (-3) ?

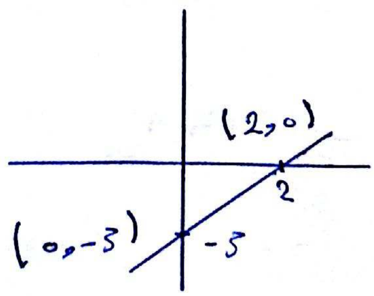
$$\boxed{y - y_1 = m(x - x_1)}$$ معادلة الخط المستقيم

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}}$$ الميل

$$m = \frac{0 - (-3)}{2 - 0} = \frac{3}{2}$$

$$y - 0 = \frac{3}{2}(x - 2)$$

$$y = \frac{3}{2}x - 3$$



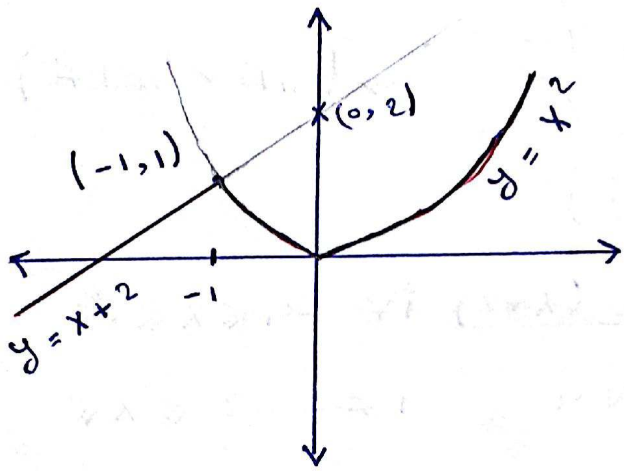
* Piecewise Function :-

الاقتران متشعب

Ex :- Find the domain and sketch the graph

$$f(x) = \begin{cases} x + 2 & ; \quad x \leq -1 \\ x^2 & ; \quad x > -1 \end{cases}$$

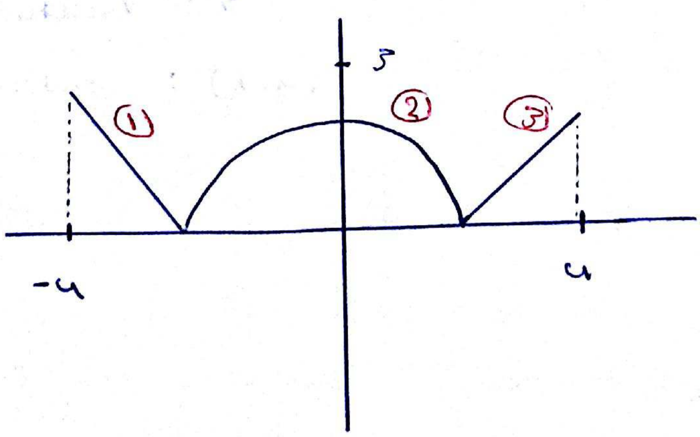
Sketch :-



اتحاد

Domain $= (-\infty, -1] \cup (-1, \infty)$

Ex : Find an expression for the function whose graph is :-



Solution :-

$$M = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{3 - 0}{-4 + 2} = \boxed{-\frac{3}{2}}$$

① $y = -\frac{3}{2}(x+2)$

② $y = \sqrt{4 - x^2}$ ⟶ (upper half)

③ $y = \frac{3}{2}(x+2)$

$$f(x) = \begin{cases} -\frac{3}{2}(x+2) & \text{if } -4 \leq x \leq -2 \\ \sqrt{4-x^2} & \text{if } -2 \leq x \leq 2 \\ \frac{3}{2}(x+2) & \text{if } 2 \leq x \leq 4 \end{cases}$$

* ملاحظة :-

معادلة الدائرة

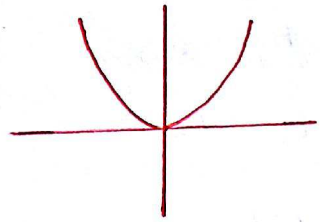
$(x-a)^2 + (y-b)^2 = r$

r : radius

(a, b) : center

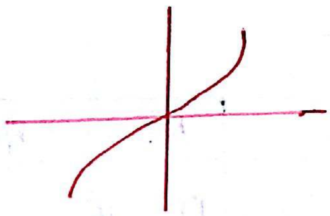
* symmetry :-

التماثل



is symmetry about y - axis

$f(x) = f(-x)$



is symmetry about origin

$f(-x) = -f(x)$

Def : * A function is called an (even) function زوجية if $f(-x) = f(x)$

* A function is called an (odd) function فردية if $f(-x) = -f(x)$

Ex :-

① $f(x) = 1 + 2x^2 + x^4$

$f(-x) = 1 + 2(-x)^2 + (-x)^4$

$= 1 + 2x^2 + x^4$

$\Rightarrow f(-x) = f(x) \longrightarrow$ even

② $f(x) = x^3 - 3x$

$f(-x) = (-x)^3 - 3(-x)$

$= -x^3 + 3x$

$= -(x^3 - 3x)$

$\Rightarrow f(-x) = -f(x) \longrightarrow$ odd

③ $f(x) = x^3 + 1$

$f(-x) = (-x)^3 + 1$

$= -x^3 + 1$

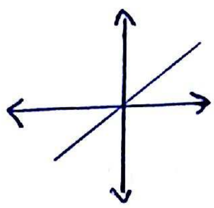
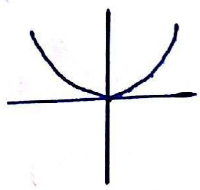
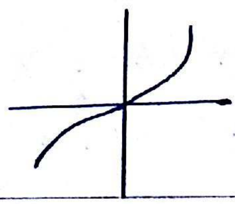
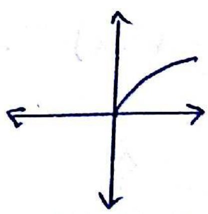
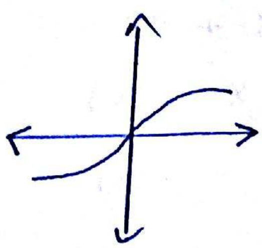
$f(-x) \neq -f(x) \rightarrow$ Not odd

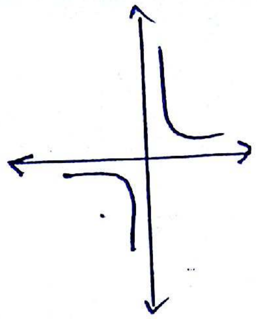
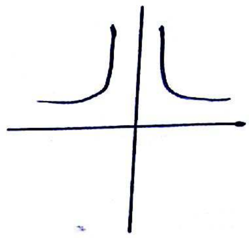
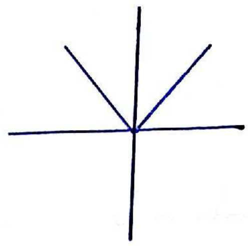
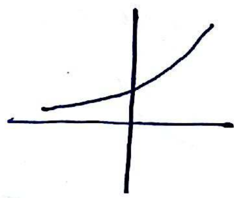
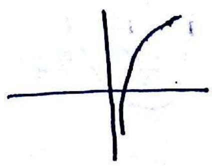
$f(-x) \neq f(x) \rightarrow$ Not even

So the function is neither .

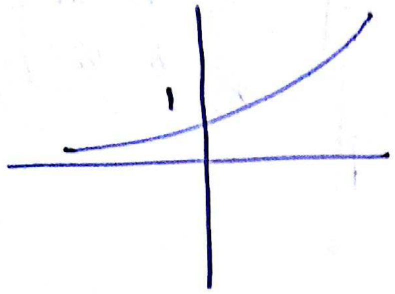
$\boxed{1.2}$ Mathematical models :-

* جدول مهم

Function	Type of fun	Simple fun	D and R	Symmetry
$f(x) = mx + b$ $(m \neq 0)$	Linear		$D = \mathbb{R}$ $R = \mathbb{R}$	about origin (odd)
$f(x) = x^2$	Power		$D = \mathbb{R}$ $R = [0, \infty)$	about y-axis (even)
$f(x) = x^3$	Power		$D = \mathbb{R}$ $R = \mathbb{R}$	about origin (odd)
$f(x) = x^{1/2}$ or $\sqrt{x}$	Power		$D = [0, \infty)$ $R = [0, \infty)$	neither
$f(x) = x^{1/3}$ or $\sqrt[3]{x}$	Power		$D = \mathbb{R}$ $R = \mathbb{R}$	about origin (odd)

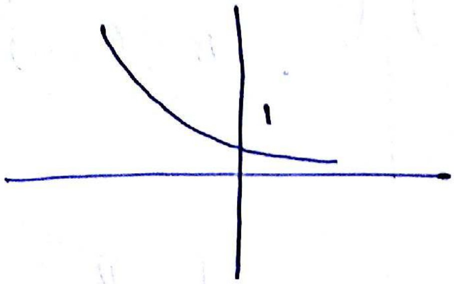
$f(x) = \frac{1}{x}$ or x^{-1}	Power		$D = \mathbb{R} - \{0\}$ $R = \mathbb{R} - \{0\}$	about origin (odd)
$f(x) = \frac{1}{x^2}$ or x^{-2}	Power		$D = \mathbb{R} - \{0\}$ $R = (0, \infty)$	about y-axis (even)
$f(x) = \lvert x \rvert$	absolute value		$D = \mathbb{R}$ $R = [0, \infty)$	about y-axis (even)
$f(x) = a^x$	Exponential		$D = \mathbb{R}$ $R = (0, \infty)$	neither
$f(x) = \log_a x$	Logarithmic		$D = (0, \infty)$ $R = \mathbb{R}$	Neither

Note :-



a^x , $a > 1$

(متزايد)



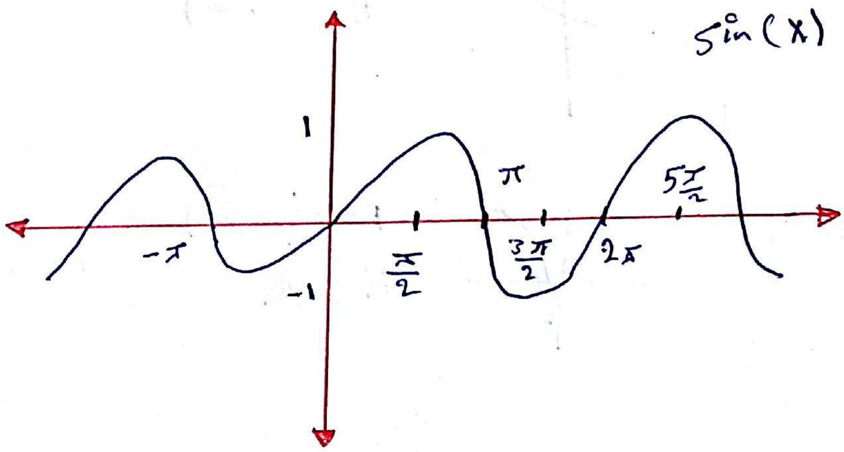
a^x , $0 < a < 1$

(متناقص)

* Trigonometric function :-

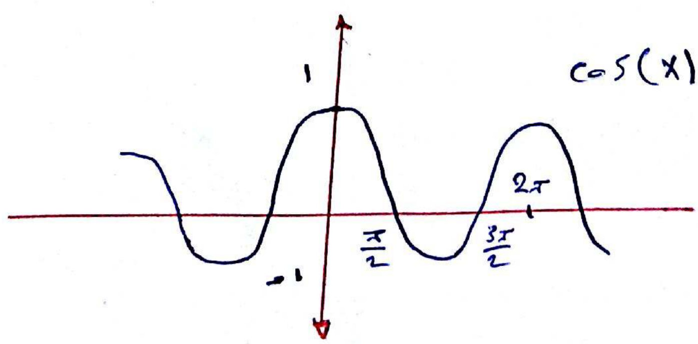
« الاقترانات المثلثية »

$\sin(x)$

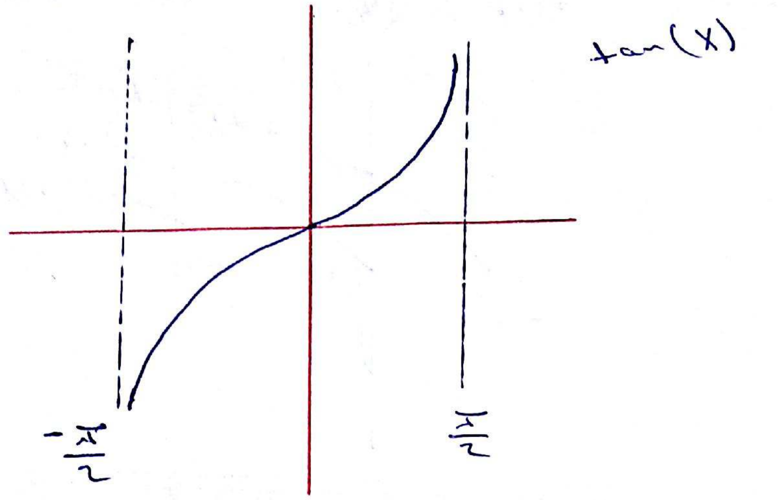


$\sin(x)$ is Periodic with Period $= 2\pi$

$\cos(x)$



$\cos(x)$ is Periodic with Period $= 2\pi$



tan(x) is Perodic with Period $= \pi$

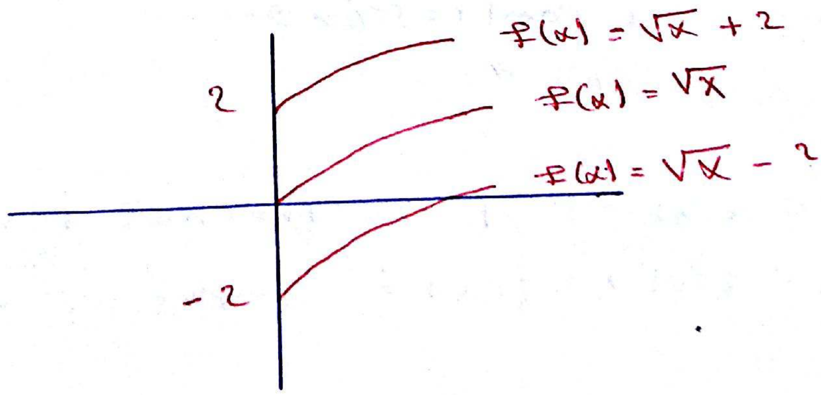
1) $\tan(x) = \mathbb{R} - \left\{ \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots \right\}$

New functions from old functions :-

* $f(x) + c \longrightarrow$ f(x) shifted up by c units
* $f(x) - c \longrightarrow$ f(x) shifted down by c units
* $f(x+c) \longrightarrow$ f(x) shifted to the left by c units
* $f(x-c) \longrightarrow$ f(x) shifted to the right by c units.

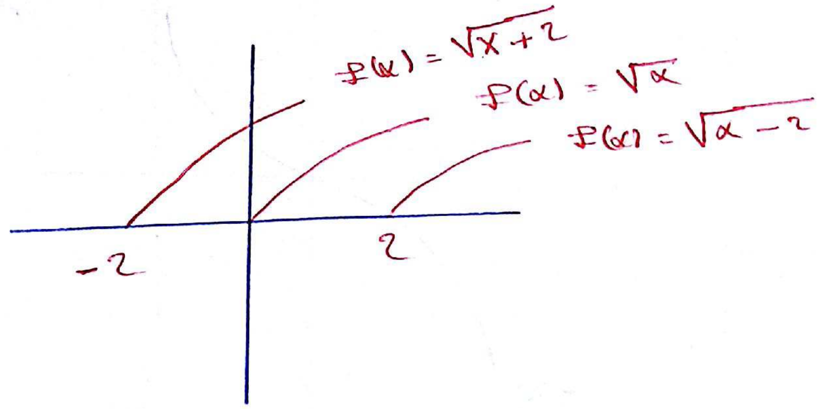
Ex: Sketch : $f(x) = \sqrt{x} + 2$ and $f(x) = \sqrt{x} - 2$

Sol:



Ex:- Sketch: $f(x) = \sqrt{x-2}$ and $f(x) = \sqrt{x+2}$

Sol:-



Reflection:-

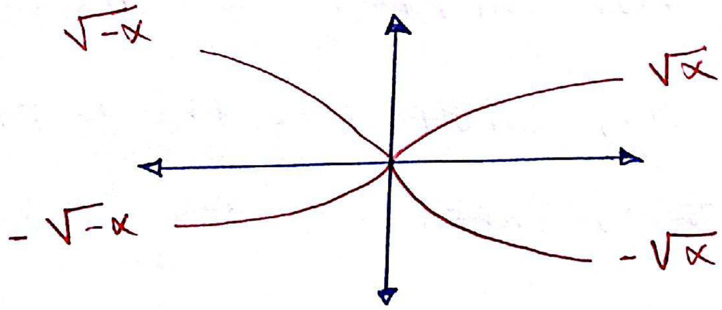
« الانعكاس »

$y = -f(x) \rightsquigarrow$ reflection about x-axis

$y = f(-x) \rightsquigarrow$ reflection about y-axis

$y = -f(-x) \rightsquigarrow$ reflection about origin

Ex: Sketch: $\sqrt{x}$, $\sqrt{-x}$, $-\sqrt{x}$, $-\sqrt{-x}$?



Stretches and compression:-

« الضغط والسحب »

$c\,f(x)$, $c > 1$; streches $f(x)$ vertically

$c\,f(x)$, $0 < c < 1$; compresses $f(x)$ vertically

$f(cx)$, $c > 1$; compresses $f(x)$ horizintally

$f(cx)$, $0 < c < 1$; stretches $f(x)$ horizintally

Sequences of transformations :-

Ex :- Explain how $f(x)$ abtained from $y = \sqrt{x}$,

$$f(x) = 2 - \sqrt{x+3} \quad ?$$

Solution :-

$$y = \sqrt{x} \xrightarrow[\text{about } x\text{-axis}]{\text{reflection}} y = -\sqrt{x}$$

then $y = -\sqrt{x} \xrightarrow[\text{Left (3) unit}]{\text{shifted to the}} y = -\sqrt{x+3}$

then $y = -\sqrt{x+3} \xrightarrow[\text{by (2) unit}]{\text{shifted up}} y = 2 - \sqrt{x+3}$

Ex : what happend to $f(x) = x^2$ to abtain

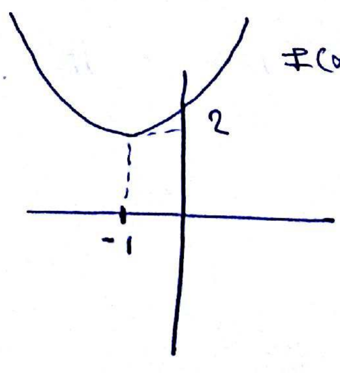
$$f(x) = x^2 + 2x + 3 \ ?$$

* use Complete the square (اكمال المربع)

Sol :- $\left(\frac{2}{2}\right)^2 = \boxed{1}$ $\qquad \leftarrow \left(\frac{x \text{ معامل}}{2}\right)^2$

③ نأخذ
معدود $\quad x^2 + 2x + 1 - 1 + 3$

$$(x+1)^2 + 2$$



$f(x) = x^2$

1 → left ① unit

2 → up ② unit

Combination of functions:-

Given $f(x)$ and $g(x)$ then;

① $(f \pm g)(x) = f(x) \pm g(x)$

$\Rightarrow D(f \pm g)(x) = D_f \cap D_g$

② $(f \cdot g)(x) = f(x) \cdot g(x)$

$\Rightarrow D(f \cdot g)(x) = D_f \cap D_g$

③ $(f/g)(x) = \dfrac{f(x)}{g(x)}$

اصفار المقام

$\Rightarrow D = (D_f \cap D_g) - \{\text{Zero of denominator}\}$

④ $(f \circ g)(x) = f(g(x)) \equiv (f) \text{ after } (g)$

$\Rightarrow D(f \circ g) = D_f$ (الجواب) $\cap$ D_g (الداخل)

EX:- $f(x) = x^2 + 3$, $g(x) = -\frac{1}{x}$, find:-

$D(f+g)$

Sol:- $D(f+g) = D_f \cap D_g$

تقاطع ، ناخذ الاشياء المشتركة

$D_f = \mathbb{R}$

$D_g = \mathbb{R} - \{0\}$

$D_f \cap D_g = \mathbb{R} - \{0\}$

Ex: $f(x) = 1+\sqrt{x-2}$, $g(x) = x-3$, find $D(f/g)$?

Sol: $D_f = [2, \infty)$

$D_g = \mathbb{R}$

$\Rightarrow D(f/g) = (D_f \cap D_g) - \{\text{اصفار المقام}\}$

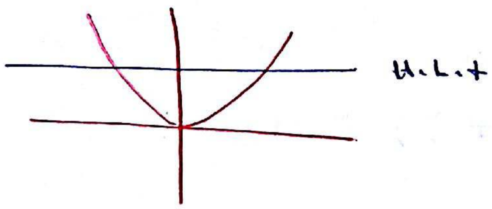
$= ([2, \infty) \cap \mathbb{R}) - \{3\}$

$[2, \infty) - \{3\}$

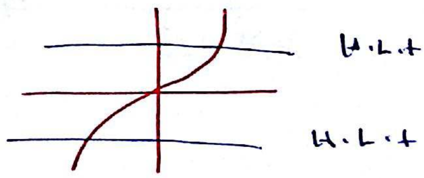
Inverse and logorithms :-

* Horizintal Line test :

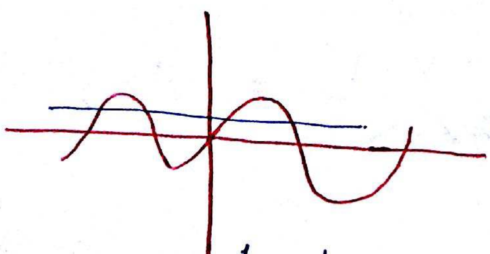
if the H.L.t cuts the curve in only one point , then it's one to one .



(Not one to one)



(one to one)



(Not one to one)

Note :-

* $D\ f^{-1}(x) = R\ f(x)$

* $R\ f^{-1}(x) = D\ f(x)$

* $f^{-1}(x)$ is reflection for $f(x)$ about $y = x$

* $(f \circ f^{-1})(x) = x$

$(f^{-1} \circ f)(x) = x$

Ex:- Find a formula for the inverse :

$$f(x) = \frac{4x-1}{2x+3}$$

Sol :-

١- بدل y مع x

٢- اجعل y موضوعاً للقانون

$$y = \frac{4x-1}{2x+3}$$

$$x = \frac{4y-1}{2y+3} \Rightarrow x(2y+3) = 4y-1$$

$$2xy + 3x = 4y - 1$$

$$\frac{y\cancel{(2x-4)}}{\cancel{2x-4}} = \frac{-3x-1}{2x-4}$$

$$y = \frac{-3x-1}{2x-4} = f^{-1}(x)$$

logarithms :-

$\log_b a = c$ mens $b^c = a$

Remark: $\log_b a = \frac{\ln a}{\ln b}$

Ex: Solve each equation for X :-

① $2^{X-5} = 3$

$\frac{(x-5)\cancel{\ln 2}}{\cancel{\ln 2}} = \frac{\ln 3}{\ln 2}$ دخل ln ع الطرفين

تذكر من قواعد ln

$x - 5 = \frac{\ln 3}{\ln 2}$ $\ln^{x} = x\ln$

$x = \frac{\ln 3}{\ln 2} + 5$

② $\ln X + \ln(X-1) = 1$

$\ln x(x-1) = 1$ دخل e ع الطرفين

$e^{\ln x(x-1)} = e^{1}$ لأن (e) تلغي عمل ln

$x(x-1) = e$

$x^2 - X - e = 0$

$$\boxed{X = \frac{1+\sqrt{1+4e}}{2}}$$ # القانون العام

only because $D_f = X > 1$

Note: $\arccos(x) = \cos^{-1}(x)$

(inverse) تعني

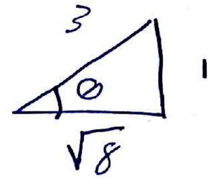
Ex: ① $\cancel{\sin}(\cancel{\sin}^{-1}\frac{1}{3}) = \frac{1}{3}$

② $\cos(\underbrace{\sin^{-1}\frac{1}{3}}_{\theta}) =$

$\theta = \sin^{-1}\frac{1}{3}$

$\sin\theta = \frac{1}{3}$

$\cos(\theta) = \frac{\sqrt{8}}{3}$

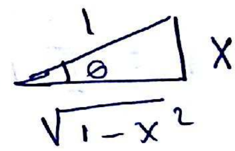


③ $\tan(\underbrace{\sin^{-1}x}_{\theta}) =$

$\theta = \sin^{-1}x$

$\sin\theta = x$

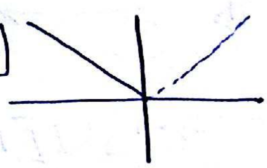
$\tan\theta = \frac{x}{\sqrt{1-x^2}}$



④ $\sin^{-1}(\sin\frac{2\pi}{3})$

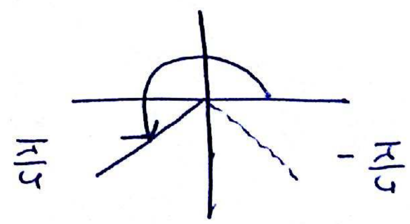
$\sin^{-1}(\sin\frac{\pi}{3}) = \frac{\pi}{3}$

$\pi - \frac{2\pi}{3} = \boxed{\frac{\pi}{3}}$



⑤ $\sin^{-1}(\sin\frac{5\pi}{4}) =$

$\sin^{-1}(\sin -\frac{\pi}{4}) = -\frac{\pi}{4}$



$\pi - \frac{5\pi}{4} = -\frac{\pi}{4}$

Trigonometric Identities :-

« المتطابقات المثلثية »

حفظ :)

1. $\cos^2(x) + \sin^2(x) = 1$
2. $\sec^2(x) = \tan^2(x) + 1$
3. $\csc^2(\alpha) = \cot^2(\alpha) + 1$
4. $\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$
5. $\cos(\alpha \pm y) = \cos x \cos y \mp \sin x \sin y$ (أطلب إشارة)
6. $\sin 2x = 2\sin x \cos x$
7. $\cos 2x = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$.

Inverse trigonometric functions :-

$$\qquad\qquad D_{f^{-1}} \qquad\qquad\qquad R_{f^{-1}}$$

حفظ

1. $\sin^{-1}(x) : [-1, 1] \longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$
2. $\cos^{-1}(x) : [-1, 1] \longrightarrow [0, \pi]$
3. $\tan^{-1}(\alpha) : \mathbb{R} \longrightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$
4. $\cot^{-1}(\alpha) : \mathbb{R} \longrightarrow (0, \pi)$
5. $\sec^{-1}(\alpha) : (-\infty, -1] \cup [1, \infty) \longrightarrow [0, \frac{\pi}{2}) \cup [\pi, \frac{3\pi}{2})$

EX: $\sin\left(\underbrace{\cot^{-1}\frac{-4}{5}}_{\alpha} + \underbrace{\csc^{-1}\frac{-9}{4}}_{\beta}\right)$?

$\alpha = \cot^{-1}\frac{-4}{5}$

$\beta = \csc^{-1}\frac{-9}{4}$

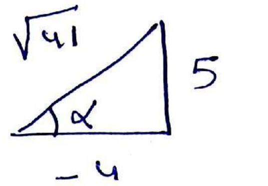
Solution

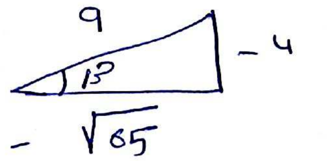
$$\Rightarrow \sin(\alpha+\beta)$$

$$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \frac{5}{\sqrt{41}} * \frac{-\sqrt{65}}{9} + \frac{-4}{\sqrt{41}} * \frac{-4}{9}$$

$$= \frac{-5\sqrt{65}+16}{9\sqrt{41}}$$





EX: (مهم) Find Domain $3\sin^{-1}(2X-1)+5$ → الزاوية $\in D$

نخص $\sin^{-1}$ بـ $[-1, 1]$

وفقط نخص اللي جوا (اللي توه) $\in D$

$$-1 \le 2X-1 \le 1$$

$$0 \le 2X \le 2$$

$$0 \le X \le 1 \quad \rightarrow D = [0,1]$$

EX: Find Range $2\sin^{-1}(3X+1)+\pi$

$$R \ \sin^{-1}X = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\left(-\frac{\pi}{2} \le \sin^{-1}(3X+1) \le \frac{\pi}{2}\right) * 2$$

$$\left(-\pi \le 2\sin^{-1}(3X+1) \le 2\right) + \pi$$

$$0 \le 2\sin^{-1}(3X+1)+\pi \le 2\pi$$

$$\rightarrow [0, 2\pi]$$

EX: Find the inverse of the function

$$f(x) = \ln\left(x + \sqrt{x^2+1}\right)$$

Solution:

$$y = \ln\left(x + \sqrt{x^2+1}\right)$$

$$x = \ln\left(y + \sqrt{x^2+1}\right)$$

$$e^x = y^2 + \sqrt{y^2+1}$$

$$(e^x - y)^2 = y^2+1$$

$$e^{2x} - 2e^x y + \cancel{y^2} = \cancel{y^2} + 1$$

$$e^{2x} - 2e^x y - 1 = 0$$

$$e^x - e^{-x} = 2y \Rightarrow y = \frac{e^x - e^{-x}}{2} \quad f^{-1}(x)$$

chapter

"2"

* Limits and Derivatives :-

Def :- $\lim_{x \to a} f(x) = L$; means if (x) approaches (a) then $f(x)$ approaches (L).

Properties :-

[1] $\lim_{x \to a} f(x) = f(a)$

[2] $\lim_{x \to a} c\, f(x) = c \lim_{x \to a} f(x)$

[3] $\lim_{x \to a} \left(f(x) + g(x) \right) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

[4] $\lim_{x \to a} \left(f(x) \cdot g(x) \right) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

[5] $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, $\lim_{x \to a} g(x) \neq$ Zero

* Indeterminates forms :-

« الكميات المرفوضة »

$\frac{0}{0}$, $\frac{\pm\infty}{\pm\infty}$, $0 \cdot \pm\infty$, $\infty - \infty$, 1^{∞} , ∞^{0} , 0^{0}

Note :- $a^{\infty} = \infty$, if $a > 1$ $\Rightarrow$ $2^{\infty} = \infty$

$a^{\infty} = 0$, if $0 < a < 1$ $\Rightarrow$ $\left(\frac{1}{2}\right)^{\infty} =$ Zero

Note :- [1] $\frac{\text{number}}{\pm\infty} =$ Zero

[2] $\frac{0}{\text{number}} =$ Zero

[3] $\frac{\text{number}}{0} = \pm\infty$

EX :- Evaluate :-

[1] $$\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$$

خطوات الحل :-

[1] نقوم بتعويض (2) في ال Lim

[2] اذا كان الجواب احد الصيغ المرفوضة ، نقوم بالتحليل او اخراج عامل مشترك او ضرب بالمرافق .

Sol :- $$\lim_{x \to 2} \frac{(2)^2 - 4}{2 - 2} = \frac{0}{0} \,!$$

$$\Rightarrow \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$$

$$\Rightarrow \lim_{x \to 2} (x+2) = \boxed{4}$$

[2] $$\lim_{x \to 0} \frac{|x|}{x}$$

* يجب اعادة تعريف القيمة المطلقة .

$$|x| = \begin{cases} x , & x \geq 0 \\ -x , & x < 0 \end{cases}$$

$$\lim_{x \to 0^+} \frac{x}{x} = \boxed{1} \qquad \lim_{x \to 0^-} \frac{-x}{x} = \boxed{-1}$$

النهاية من اليمين
لا تساوي النهاية من اليسار
اذن النهاية غير موجودة

(doesn't exsit)

[3] $$\lim_{x \to 2} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1} \Rightarrow \text{تعويض مباشر} \Rightarrow \frac{0}{0} \,!!$$

قيمة مرفوضة

* هون تستخدم الضرب بالمرافق !

$$\lim_{x \to 2} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1} * \frac{\sqrt{6-x} + 2}{\sqrt{6-x} + 2} * \frac{\sqrt{3-x} + 1}{\sqrt{3-x} + 1}$$

$$\lim_{x \to 2} \frac{6 - x - 4}{3 - x - 1} = \lim_{x \to 2} \frac{(2-x)}{(2-x)} \cdot \lim \frac{\sqrt{3-x} + 1}{\sqrt{6-x} + 2} = 1 \cdot \frac{2}{4} = \boxed{\frac{1}{2}}$$

Ex:- Find the Limit :-

$$\lim_{x \to 2} \frac{5^x - 25}{x - 2} = ??$$

Note:
if the $\lim_{x \to a} f(x) = \frac{0}{0}, \frac{\pm\infty}{\pm\infty}$
(use L'Hopital's Rule)

قاعدة لوبيتال :-

$$\frac{\text{مشتقة البسط}}{\text{مشتقة المقام}}$$

Sol: (تعويض مباشر)

$$\frac{5^2 - 25}{2 - 2} = \frac{0}{0} \; !!$$

$$\lim_{x \to 2} \frac{5^x \ln 5}{1} = 25 \ln(5)$$

* سيتم ~~شرحها~~ شرحها بالتفصيل لقدام :D

Sandwich Theorem :-

In the neighbourhood of $x = a$

$$f(x) \le g(x) \le h(x)$$

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = \ell \text{ , then } \lim_{x \to a} g(x) = \ell$$

$$\Rightarrow \ell \le \lim_{x \to a} g(x) \le \ell$$

Ex: If $4x - 9 \le f(x) \le x^2 - 4x + 7$, find $\lim_{x \to 4} f(x)$?

Sol:-

$$\lim_{x \to 4} 4x - 9 = 7$$

$$\lim x^2 - 4x + 7 = 7$$

So $\lim_{x \to 4} f(x) = 7$ by squeezing thm

Ex :- $\lim e^{-2x} \cos x$

تعويض → اشي ما بعرفه .

$\cos x \Rightarrow$ use squeezing theorm

$$-1 \le \cos x \le 1$$

$$-e^{-2x} \le e^{-2x}\cos x \le e^{-2x}$$

$$\downarrow \qquad\qquad \downarrow \qquad\qquad \downarrow$$

$$0 \qquad\qquad 0 \qquad\qquad 0$$

Rule :- $\lim \dfrac{\sin x}{x} = 1$

$$\lim \left(1 + \frac{a}{x}\right)^x = e^a$$

vertical Asymptote:

« خط تقارب عمودي »

Def: the line $x = a$ is called a vertical asymptote of the curve $y = f(x)$ if a least of the follwing statements is true :

$$\lim_{x \to a} f(x) = \infty \qquad \lim_{x \to a^-} f(x) = \infty \qquad \lim_{x \to a^+} f(x) = \infty$$

$$\lim_{x \to a} f(x) = -\infty \qquad \lim_{x \to a^-} f(x) = -\infty \qquad \lim_{x \to a^+} f(x) = -\infty$$

EX: Find V.A

$\boxed{1}$ $f(x) = \dfrac{x-2}{x^2-4}$

$x^2 - 4 = 0$

$x^2 = 4$

$\boxed{x = \pm 2}$

Sol: $\lim\limits_{x \to 2} \dfrac{x-2}{x^2-4} = \dfrac{0}{0}$!!

$\lim\limits_{x \to 2} \dfrac{\cancel{x-2}}{\cancel{(x-2)}(x+2)} = \boxed{\dfrac{1}{4}} \to$ No V.A

$\lim\limits_{x \to -2} \dfrac{x-2}{(x^2-4)} = \dfrac{-4}{0} = \boxed{-\infty} \to$ $x = -2$ is a V.A

$\boxed{2}$ $f(x) = \dfrac{\sin(x-2)}{x-2}$

$\lim\limits_{x \to 2} \dfrac{\sin(x-2)}{x-2} = \boxed{1}$

$x - 2 = 0$

$\boxed{x = 2}$

$\to$ No V.A

$\boxed{3}$ $f(x) = \dfrac{x^2-1}{x+1}$

$x + 1 = 0$

$\boxed{x = -1}$

$\lim\limits_{x \to -1} \dfrac{x^2-1}{x+1} = \dfrac{(x-1)\cancel{(x+1)}}{\cancel{x+1}} = \boxed{-2}$

No V.A

Continuity

« الاتصال »

Def: A function f is continuous at the Point $X = a$ if the following are true:

[1] $f(a)$ is defined

[2] $\lim\limits_{X \to a} f(X)$ exists

[3] $\lim\limits_{X \to a} f(X) = f(a)$

Note: if f and g are continuous at $X = a$, then $f+g$, $f \cdot g$ and f/g ($g(a) \neq 0$) are continuous at $X = a$.

* Typs of discontinuites :-

[1] If the $\lim\limits_{X \to a} f(X) = \pm \infty$, then at (a) $\boxed{\text{infinite}}$ dis. con

[2] If $\lim\limits_{X \to a^+} f(x) \neq \lim\limits_{X \to a^-} f(X)$, then at (a) $\boxed{\text{Jump}}$ dis. con

[3] If $\lim\limits_{X \to a} f(x)$ exist but $f(a)$ isn't defind

or $f(a) \neq \lim\limits_{X \to a} f(x)$, then at (a) $\boxed{\text{Removable}}$ dis. con

EX: The value of the constant (K) that maks

$$F(x) = \begin{cases} x^2 + 4kx & , x \leq 1 \\ \dfrac{k(1-\sqrt{x})}{1-x} & , x > 1 \end{cases}$$

continuous at

$x = 1$ is ??

Sol:

$$\lim_{x \to 1^+} F(x) = \lim_{x \to 1^-} F(x)$$

$$\lim_{x \to 1^+} \frac{k(1-\sqrt{x})}{1-x} = \lim x^2 + 4kx$$

$$\lim_{x \to 1^+} \frac{k\cancel{(1-\sqrt{x})}}{\cancel{(1-\sqrt{x})}(1+\sqrt{x})} = 1 + 4k$$

$$\frac{k}{1+1} = 1 + 4k$$

$$\frac{k}{2} = 1 + 4k$$

\# $\boxed{k = \frac{-2}{7}}$

EX: the discontinuities of $F(x) = \dfrac{4}{5 + e^{\left(\frac{1}{x-7}\right)}}$

is ??

at $x = 7$

EX:- Find interval of cont.? ⇐ * ملاحظة اذا طلب السؤال فترات الاتصال !

$F(x) = \sqrt{x+2}$

① ⇒ Find the domain

② ⇒ open interval

$D = [-2, \infty) \Rightarrow$ cont. interval $= (-2, \infty)$

limits at infinity (Horizontal Asymptote)

Def :- Let F be a function defined on same interval, then

$$\boxed{\lim_{x\to\infty} f(x) = L} \qquad \boxed{\lim_{x\to -\infty} f(x) = L}$$

* **Note :** (Rule)

$$\lim_{x\to\pm\infty} \frac{Poly}{Poly} =$$

① صفر اذا كانت درجة البسط < درجة المقام

② $\frac{\text{معامل اكبر أس}}{\text{معامل اكبر أس}}$ اذا كانت درجة البسط = درجة المقام

③ $\pm\infty$ اذا كانت درجة البسط > درجة المقام .

EX : find H.A for $f(x) = \dfrac{\sqrt{3x^2+1}}{3x-1}$?

Sol :

⇐ نأخذ النهاية من يمين ويسار الـ ∞ ⇒

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\sqrt{x^2\left(3+\frac{1}{x^2}\right)}}{x\left(3-\frac{1}{x}\right)}$$ (عامل مشترك)

$$= \lim_{x\to\infty} \frac{|x|\sqrt{3+\frac{1}{x^2}}}{x\left(3-\frac{1}{x}\right)}$$

$$= \cancel{\lim_{x\to\infty}} \frac{\sqrt{3+\frac{1}{\infty}}}{3-\frac{1}{\infty}} = \frac{\sqrt{3}}{3} \text{ is H.A}$$

$$\lim_{x \to -\infty} \frac{-x\sqrt{3-\frac{1}{x^2}}}{x(3-\frac{1}{x})} = \frac{-\sqrt{3}}{3} \text{ is H.A}$$

$$\underbrace{\boxed{y = \frac{\sqrt{3}}{3}}, \boxed{y = -\frac{\sqrt{3}}{3}}}_{\text{H.A}}$$

Ex: Find H.A $f(x) = \dfrac{x+1}{x^2+4x}$?

Sol: $\lim_{x \to \infty} \dfrac{x+1}{x^2+4x}$ $\begin{matrix} \rightarrow \text{Poly} \\ \rightarrow \text{Poly} \end{matrix}$

So $\lim_{x \to \infty} \dfrac{x}{x^2} = \dfrac{1}{\infty} = \text{zero}$

Ex: Find H.A $f(x) = \tan^{-1}(x)$?

Sol:- $\lim_{x \to \infty} \tan^{-1}(x) = \tan^{-1}(\infty) = \boxed{\frac{\pi}{2}}$

$\lim_{x \to -\infty} \tan^{-1}(x) = \tan^{-1}(-\infty) = \boxed{-\frac{\pi}{2}}$

$$\underbrace{\boxed{y = \frac{\pi}{2}}, \boxed{y = -\frac{\pi}{2}}}_{\text{H.A}}$$

Note:

* $\tan^{-1}(\infty) = \frac{\pi}{2}$
* $\tan^{-1}(-\infty) = -\frac{\pi}{2}$

$\lim_{X \to a} \frac{f(x) - f(a)}{X - a} \Rightarrow f'(x)$

المشتقة الأولى ⇐

EX: $f(x) = \sqrt{X}$ find $f'(X)$?

$\Rightarrow$ باستخدام تعريف المشتقة $\Rightarrow$

$$f'(X) = \lim_{X \to a} \frac{f(x) - f(a)}{X - a}$$

$$= \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{X - a}$$

$$= \lim \frac{\sqrt{x} - \sqrt{a}}{X - a} * \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}}$$

ضرب مرافق

$$= \lim \frac{\cancel{X - a}}{\cancel{X - a} * \sqrt{x} + \sqrt{a}}$$

$$= \boxed{\frac{1}{2\sqrt{a}}}$$

Chapter

“3”

Derivatives :-

Rules of Derivatives:

Function	Derivatives
C (ثابت)	zero
X^n	nX^{n-1} (for all n)
$e^{f(X)}$	$f'(X) \cdot e^{f(X)} \cdot \ln e$ "Note: $\ln e = 1$"
$a^{f(X)}$	$f'(X) \cdot a^{f(\alpha)} \cdot \ln a$
$\ln f(\alpha)$	$\dfrac{f'(X)}{f(X)}$ $\quad$ مشتقة ما داخل ln / نفسه
$f(\alpha) \pm g(\alpha)$	$f'(X) \pm g'(\alpha)$
$f(\alpha) \cdot g(\alpha)$	$f(\alpha) \cdot g'(\alpha) + g(\alpha) \cdot f'(\alpha)$
$\dfrac{f(\alpha)}{g(\alpha)}$	$\dfrac{g(\alpha) \cdot f'(\alpha) - f(\alpha) \cdot g'(\alpha)}{(g(\alpha))^2}$

<< الاقترانات المثلثية >>

$\sin(X)$	$\cos(x)$
$\cos(x)$	$-\sin(\alpha)$
$\tan(\alpha)$	$\sec^2(\alpha)$
$\cot(\alpha)$	$-\csc^2(X)$
$\sec(X)$	$\sec(X) \cdot \tan(\alpha)$
$\csc(X)$	$-\csc(X) \cdot \cot(\alpha)$

Ex: find $f'(x)$:-

$\boxed{1}$ $2x^{100} + \sqrt{x} + 4$

$\Rightarrow 200x^{99} + \frac{1}{2\sqrt{x}} + 0$

$\boxed{2}$ $e^{x^2} + 3^{x^2}$

$\Rightarrow 2x \cdot e^{x^2} \cdot \ln e + 2x \cdot 3^{x^2} \cdot \ln 3$

$\boxed{3}$ $\sin^{99}(x)$

Note: $\sin(x) \Rightarrow$ يعيد نفسه

كل 4 مرات

$\Rightarrow 4\overline{)99} \Rightarrow$ باقي $\Rightarrow \boxed{3}$

اذن الجواب هو المشتقة $\boxed{3}$ لـ $\sin(x)$

$\sin^{(99)} \Rightarrow \sin^{(3)}$

$\boxed{4}$ if $y = \cos 3x$ then $\frac{dy^{42}}{dx^{42}}$

$\Rightarrow 4\overline{)42} \Rightarrow$ الباقي $= \boxed{2}$

إذن المشتقة الثانية

$\Rightarrow (3)^{42} \cdot -\cos(3x)$

[5] if $f(x) = 2^x$ find $\frac{dy^{100}}{dx^{100}}$?

$\Rightarrow$ $f'(x) = 2^x \cdot \ln 2$

$f''(x) = 2^x \cdot \ln 2 \cdot \ln 2 = 2^x (\ln 2)^2$

$f'''(x) = 2^x \cdot \ln 2 \cdot \ln 2 \cdot \ln 2 = 2^x (\ln 2)^3$

$\Rightarrow$ $\frac{dy^{100}}{dx^{100}} = 2^x \cdot (\ln 2)^{100}$

L'Hopital's Rule :-

If $\lim\limits_{x \to c} \frac{f(x)}{g(x)}$ Produces an indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

EX: ① $\lim\limits_{x \to \infty} \frac{x}{e^x}$? $= \frac{\infty}{\infty}$!!

$\Rightarrow \lim\limits_{x \to \infty} \frac{1}{e^x} = \frac{1}{\infty} = 0$ by L'Hop Rule

② $\lim\limits_{x \to 0^+} \frac{\ln x}{\csc x}$? $= \frac{-\infty}{\infty}$!

$\lim\limits_{x \to 0^+} \frac{\frac{1}{x}}{-\csc x \cdot \cot x}$ $\Rightarrow \lim\limits_{x \to 0^+} \frac{-1}{\csc x \cdot \cot x \cdot x}$

$\Rightarrow \lim\limits_{x \to 0^+} \frac{-\sin x \tan x}{x} = -1 * 0 = 0$

Continuity :-

ملاحظة :-

شروط الاشتقاق :

① المشتقة من اليمين = المشتقة من اليسار

② الاقتران متصل

EX: Find (m) and (b) that make $f(x)$ differentiable everywhere ?

$$f(x) = \begin{cases} x^2 , & x \leq 2 \\ mx+b , & x > 2 \end{cases}$$

Sol: بما أن الاقتران قابل للاشتقاق اذن هو متصل

يعني النهاية من اليمين = النهاية من ~~اليسار~~ اليسار

$$2x \Big|_{x=2} = m \quad \Rightarrow \boxed{m = 4}$$

then, $\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^-} f(x)$

$$\Rightarrow \quad 2m + b = 4$$

$$2(4) + b = 4$$

$$\boxed{b = -4}$$

Chain Rule

<< قاعدة السلسلة >>

(f)	(f')
$f(g(x))$	$f' g(x) \cdot g'(x)$

$y = f(u)$
$u = g(x)$ $\Rightarrow y' = \frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx}$

Ex:- $f(x) = \tan(\sin(x^3))$, find $f'(x)$?

Sol:- $f'(x) = \sec^2(\sin(x^3)) \cdot \cos(x^3) \cdot 3x^2$

Ex: $f(x) = e^{\sec(3x)}$

Sol:- $f'(x) = 3 \cdot \sec(3x) \cdot \tan(3x) \cdot e^{\sec(3x)}$

Ex:- If $y = \sin(u)$, $u = x^2$

find $\frac{dy}{dx}$?

Chain Rule
ثلاث متغيرات
x, y, u

Sol: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$\Rightarrow \cos u \cdot 2x$

$= 2x \cdot \cos(x^2)$

Implicit differentiation :-

<< الاشتقاق الضمني >>

$y = \sin(xy) \Rightarrow$ ضمني

$y = \sin(x) \Rightarrow$ صريح

EX: $\sin(xy) = x^2$, find $\frac{dy}{dx}$?

Sol: $\cos(xy) * (xy' + y \cdot 1) = 2x$

$$xy' + y = \frac{2x}{\cos xy}$$

$$y' = \frac{dy}{dx} = \frac{2x}{\cos xy * x} - y$$

EX: $x^3 + y^3 = 6x \cdot y$, find y' ?

Sol: $3x^2 + 3y^2 \cdot y' = 6x \cdot y' + 6y$

$$3y^2 y' - 6x y' = 6y - 3x^2$$

$$y'(3y^2 - 6x) = 6y - 3x^2$$

$$y' = \frac{6y - 3x^2}{3y^2 - 6x}$$

! << 100% >>

$f(x)$	$f'(x)$
$\sin^{-1}(x)$	$\dfrac{1}{\sqrt{1-x^2}} \cdot 1$
$\cos^{-1}(x)$	$\dfrac{-1}{\sqrt{1-x^2}} \cdot 1$
$\tan^{-1}(x)$	$\dfrac{1}{1+x^2} \cdot 1$
$\cot^{-1}(x)$	$\dfrac{-1}{1+x^2} \cdot 1$
$\sec^{-1}(x)$	$\dfrac{1}{x\sqrt{x^2-1}} \cdot 1$
$\csc^{-1}(x)$	$\dfrac{-1}{x\sqrt{x^2-1}} \cdot 1$

Ex:- if $y = \sin^{-1}(x)$, Prove that $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

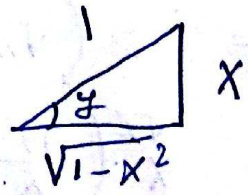
Sol:- $\sin y = x$ $\Rightarrow$ ركب sin للطرفين

$\cancel{\sin}\ \cancel{\sin^{-1}}$

$\Rightarrow \cos y \dfrac{dy}{dx} = 1$ $\Rightarrow$ اشتق الطرفين

<< ضمنياً ٢٣

$\dfrac{dy}{dx} = \dfrac{1}{\cos y}$



\# $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

EX :- if $y = (\sin^{-1} x^3)^2$, find y' ?

Sol :- $y' = 2(\sin^{-1} x^3) \cdot \dfrac{1}{\sqrt{1-(x^3)^2}} \cdot 3x^2$

EX : if $y = \cos^{-1}(e^x)$, find $\dfrac{dy}{dx}$?

Sol :- $\dfrac{dy}{dx} = \dfrac{-1}{\sqrt{1-(e^x)^2}} \cdot e^x = \dfrac{-e^x}{\sqrt{1-e^{2x}}}$

Rule :- $\dfrac{d}{dx} \log_a f(x) = \dfrac{f'(x)}{f(x) \ln a}$

EX :- if $y = \log_2 (x^3+1)$, find $\dfrac{dy}{dx}$?

Sol :- $\dfrac{dy}{dx} = \dfrac{3x^2}{(x^3+1) \cdot \ln 2}$

مهم

$\boxed{\text{EX :-}}$ if $y = x^{\sqrt{x}}$, find $\dfrac{dy}{dx}$?

Note :- $x^x \Rightarrow$ use ln

Sol :- $\ln y = \ln x^{\sqrt{x}}$

تذكر : ⇐

$\ln x^{b} \to b \ln x$

$\dfrac{1}{y} \dfrac{dy}{dx} = \sqrt{x} \cdot \dfrac{1}{x} + \dfrac{1}{2\sqrt{x}} \cdot \ln x$

$\Rightarrow \dfrac{dy}{dx} = y\left(\sqrt{x} \cdot \dfrac{1}{x} + \dfrac{1}{2\sqrt{x}} \cdot \ln x\right)$

ارجع قيمة y

$= x^{\sqrt{x}}\left(\dfrac{\sqrt{x}}{x} + \dfrac{\ln x}{2\sqrt{x}}\right)$

Remark :-

[1] Tangent line is "Horizontal" if :-

$$\frac{dy}{dx} = \text{zero} \quad (dy = \text{zero})$$ البسط = صفر

[2] Tangent line is "vertical" if :-

$$\frac{dy}{dx} = \pm\infty \quad (dx = \pm\infty)$$ المقام = صفر

EX: if $y = (\sin 3x)^x$, then $\frac{dy}{dx} = ??$

Sol:- $\ln y = x \ln(\sin 3x)$

$$\frac{1}{y}\frac{dy}{dx} = \frac{3x \cdot \cos(3x)}{\sin(3x)} + \ln \sin(3x) \cdot 1$$

$$\frac{dy}{dx} = y\left(\frac{3x \cdot \cos(3x)}{\sin(3x)} + \ln \sin(3x)\right)$$

$$\frac{dy}{dx} = \sin(3x)^x\left(\frac{3x \cdot \cos(3x)}{\sin(3x)} + \ln \sin(3x)\right)$$

 # Rule :- $$\left(f^{-1}(x)\right)' = \frac{1}{f'\left(f^{-1}(x)\right)}$$

Ex:- if $f(4) = 5$, $f'(4) = \frac{2}{3}$, find $(f^{-1}(5))'$?

Sol:- $$(f^{-1}(5))' = \frac{1}{f'(f^{-1}(5))} = \frac{1}{f'(4)} = \frac{1}{2/3} = \boxed{\frac{3}{2}}$$

EX :- if $f(x) = e^{x} \cos x$, $(0,1)$

find tangent line at $(0,1)$?

Sol :- $f'(x) = e^{x} * -\sin x + \cos x \, e^{x}$

$f'(0) = m = 1 * -0 + 1 = \boxed{1}$

$\Rightarrow y - 1 = 1(x - 0)$

$y - 1 = x$

$y = x + 1$

Note :- $M_{normal} = 1 \Rightarrow M_{tangent} = -1$

« ميل العمودي على المماس = سالب مقلوب المماس »

EX :- ~~if $y = x^2$~~

find equ. of tangent for :

$x^2 + 2xy - y^2 + x = 2$ at $(1, 2)$?

Sol $\Rightarrow$ $2x + \left(2x \frac{dy}{dx} + 2y\right) - 2y \frac{dy}{dx} + 1 = 0$

$2(1) + \left(2(1) \frac{dy}{dx} + 2(2)\right) - 2*2 \frac{dy}{dx} + 1 = 0$

$\frac{dy}{dx} = \boxed{\frac{7}{2}}$

equ : $y - 2 = \frac{7}{2}(x - 1)$

linear approximation

$$f(x) \approx f(a) + f'(a)(x-a)$$

⊲= يستخدم لتقريب إقتران معين =⊲

EX :-

$f(x) = (1+x)^{15}$, find approximate lineary at $a =$ zero ?

Sol :-

$$f'(x) = 15(1+x)^{14} \cdot 1 \longrightarrow f'(0) = 15$$

$$f(x) = f(0) + f'(0)(x-0)$$

$$\Rightarrow f(x) \approx 1 + 15x \text{ , at } a = \text{zero} .$$

EX :-

approximate $\sin(59^\circ)$?

To approximate value $\Rightarrow$

Sol :

$$f(x) = \sin x \longrightarrow f'(x) = \cos x$$

$$x = 59^\circ \quad , \quad a = 60^\circ$$

$$f(a) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$f'(a) = \cos 60^\circ = \frac{1}{2}$$

$$\sin(59^\circ) \approx \frac{\sqrt{3}}{2} + \frac{1}{2}(59^\circ - 60^\circ)$$

$$\approx \frac{\sqrt{3}}{2} - \frac{1}{2}(1^\circ)$$

$$\approx \frac{\sqrt{3}}{2} - \frac{1}{2}\left(\frac{\pi}{180}\right)$$

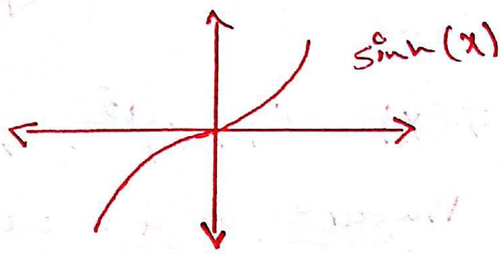
note :-

$$1^\circ = \frac{\pi}{180}$$

Hyperbolic functions:-

[1] Hyp $\sin x \Rightarrow \sinh x = \dfrac{e^{x} - e^{-x}}{2}$

odd function



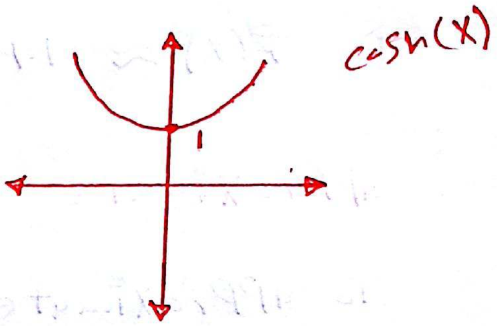
$D \sinh x = \mathbb{R}$

$R \sinh x = \mathbb{R}$

---

[2] Hyp $\cos x \Rightarrow \cosh x = \dfrac{e^{x} + e^{-x}}{2}$

even function



$D \cosh x = \mathbb{R}$

$R \cosh x = [1, \infty)$

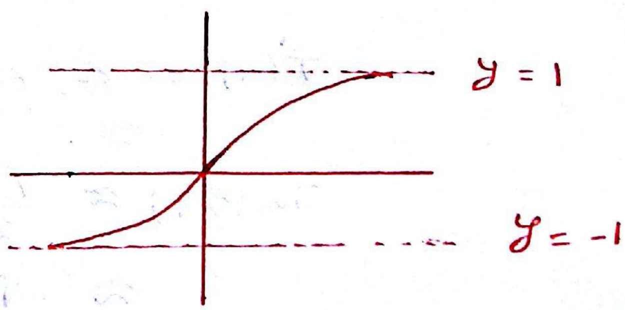
---

[3] Hyp $\tan x \Rightarrow \tanh(x) = \dfrac{\sinh x}{\cosh x} = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

odd function

$D \tanh(x) = \mathbb{R}$

$R \tanh(x) = (-1, 1)$



(المقام) $e^{x} + e^{-x} \neq 0$ (No V.A)

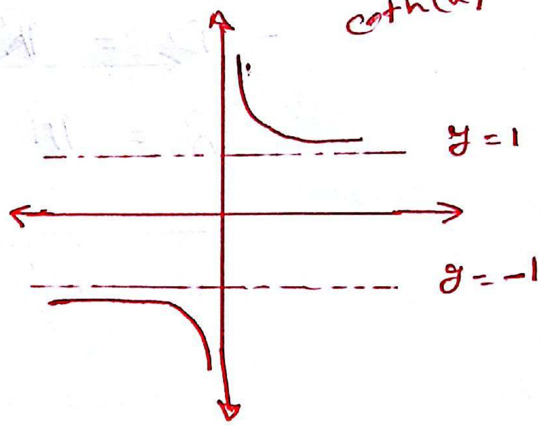
$\lim_{x \to \pm\infty} \left(\dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \right) = \begin{matrix} y = 1 \\ y = -1 \end{matrix} \Big\} \text{ H.A}$

4. Hyp cot X $\Rightarrow$ coth(X) $= \dfrac{\cosh X}{\sinh X}$

$$= \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$$

- odd function
- $D_{\coth x} = \mathbb{R} - \{0\}$
- $R = (-\infty,-1) \cup (1,\infty)$

  or $R - [-1,1]$



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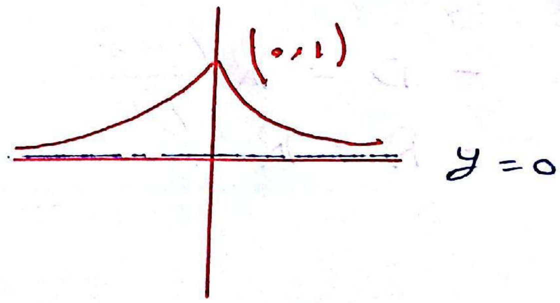
5. Hyp sec X $\Rightarrow$ sech X $= \dfrac{1}{\cosh(x)}$

$$= \frac{2}{e^{x}+e^{-x}}$$

- even function
- $D_{\text{sech}\,x} = \mathbb{R}$
- $R = (0,1]$



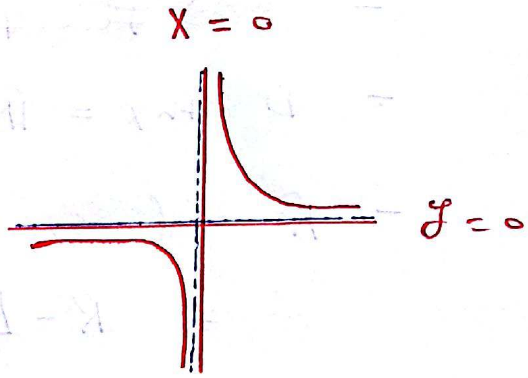
$e^{x}+e^{-x} \neq 0$ (No V.A)

$$\lim_{x \to \pm\infty} \left(\frac{2}{e^{x}+e^{-x}} \right) = \frac{2}{\infty} = 0$$

6. Hyp csc x $\Rightarrow$ $\operatorname{csch} x = \dfrac{1}{\sinh(x)}$

$$= \frac{2}{e^{x} - e^{-x}}$$

- odd function
- $D = \mathbb{R} - \{0\}$
- $R = \mathbb{R} - \{0\}$



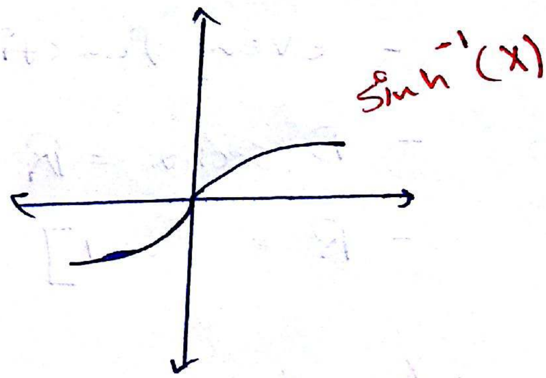
$X = 0$ (V.A)

$y = 0$ (H.A)

Inverse Hyperbolic functions :-

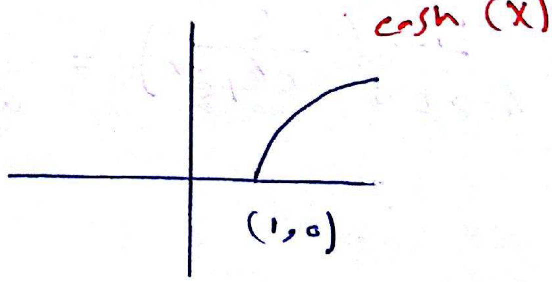
1. $\sinh^{-1}(x)$

- $D = \mathbb{R}$
- $R = \mathbb{R}$



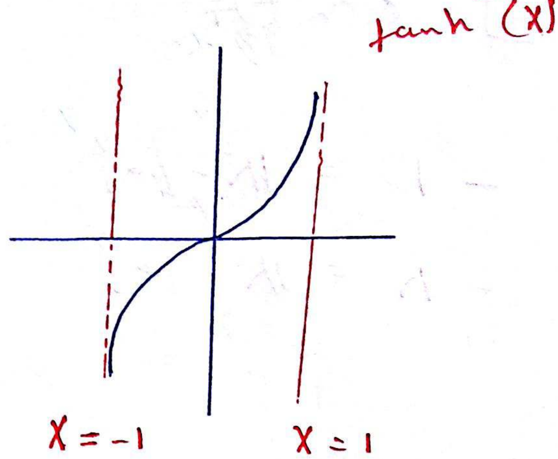
2. $\cosh^{-1}(x)$

- $D = [1, \infty)$
- $R = [0, \infty)$



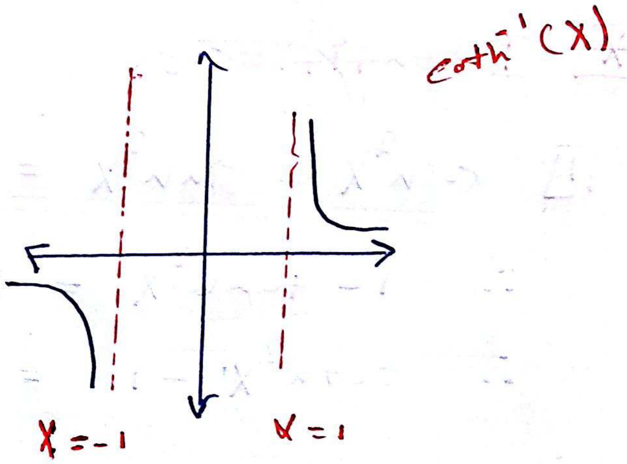
$\boxed{3}$ $\tanh^{-1}(x)$

- $D = (-1, 1)$
- $R = \mathbb{R}$



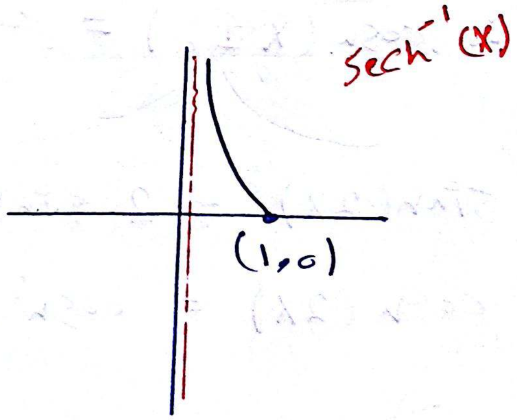
$\boxed{4}$ $\coth^{-1}(x)$

- $D = (-\infty, -1) \cup (1, \infty)$
- $R = \mathbb{R} - \{0\}$



$\boxed{5}$ $\operatorname{sech}^{-1}(x)$

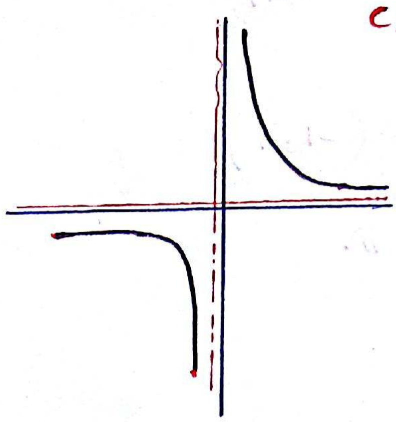
- $D = (0, 1]$
- $\mathbb{R} = [0, \infty)$



$\boxed{6}$ $\operatorname{csch}^{-1}(x)$

- $D = \mathbb{R} - \{0\}$
- $R = \mathbb{R} - \{0\}$

$\operatorname{csch}^{-1}(x)$



$\boxed{*}$ Identities :-

حفظ

$\boxed{1}$ $\cosh^2 x - \sinh^2 x = 1$

$\therefore \; 1 - \tanh^2 x = \operatorname{sech}^2 x$

$\therefore \; \coth^2 x - 1 = \operatorname{csch}^2 x$

$\boxed{2}$ $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$

$\therefore \; \cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$

$\boxed{3}$ $\sinh(2x) = 2 \sinh x \cosh x$

$\cosh(2x) = \cosh^2 x + \sinh^2 x$

$\boxed{4}$ $\cosh^2(x) = \dfrac{\cosh(2x) + 1}{2}$

$\sinh^2(x) = \dfrac{\cosh(2x) - 1}{2}$

(*) $\cosh x + \sinh x = e^{x}$

(*) $\cosh x - \sinh x = e^{-x}$

(*) $\dfrac{1 + \tanh x}{1 - \tanh x} = e^{2x}$

Ex :- Prove that :-

$$\cosh x + \sinh x = e^{x}$$

Sol :-

$$\cosh x + \sinh x = \frac{e^{x}+e^{-x}}{2} + \frac{e^{x}-e^{-x}}{2}$$

$$= \frac{e^{x}+\cancel{e^{-x}}+e^{x}-\cancel{e^{-x}}}{2}$$

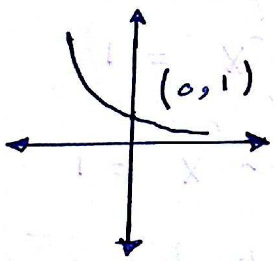
$$= \frac{\cancel{2}e^{x}}{\cancel{2}} = e^{x}$$

Ex :- Sketch :-

$(\cosh x - \sinh x)^{10}$?

Sol :-

$$(\cosh x - \sinh x)^{10} = (e^{-x})^{10} = e^{-10x}$$

$\Rightarrow$ 

Ex : Prove :- $(\cosh x)^2 - (\sinh x)^2 = 1$?

Sol :-

$$= \left(\frac{e^{x}+e^{-x}}{2}\right)^2 - \left(\frac{e^{x}-e^{-x}}{2}\right)^2$$

$$= \frac{e^{2x}+2+e^{-2x}}{4} - \frac{e^{2x}-2+e^{-2x}}{4}$$

$$= \frac{e^{2x}+2+e^{-2x}-e^{2x}+2-e^{-2x}}{4} = \frac{4}{4} = \boxed{1} \#$$

EX :- If $\tanh x = \frac{-12}{13}$ find the other five Hyperbolic function ?

Sol :-

Note :-
$\operatorname{sech} x$
$\cosh x$ > 0
دائما موجبة

① $\boxed{\coth x = \frac{-13}{12}}$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$1 - \left(-\frac{12}{13}\right)^2 = \operatorname{sech}^2 x$$

$$\frac{25}{169} = \operatorname{sech}^2 x$$

$$\operatorname{sech} x = \pm\sqrt{\frac{25}{169}} = \pm \frac{5}{13} = + \frac{5}{13}$$

② $\boxed{\operatorname{sech} x = \frac{5}{13}}$

③ $\boxed{\cosh x = \frac{13}{5}}$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\left(\frac{13}{5}\right)^2 - \sinh^2 x = 1$$

$$\sinh x = \pm\sqrt{\frac{144}{25}}$$

④ $\boxed{\sinh x = -\frac{12}{5}}$

⑤ $\boxed{\operatorname{csch} x = -\frac{5}{12}}$

Derivatives :-

$f(x)$		$f'(x)$
1 $\sinh x$	$\longrightarrow$	$\cosh x$
2 $\cosh x$	$\longrightarrow$	$\sinh x$
3 $\tanh x$	$\longrightarrow$	$\text{sech}^2 x$
4 $\coth x$	$\longrightarrow$	$-\text{csch}^2 x$
5 $\text{sech}\, x$	$\longrightarrow$	$-\text{sech}\, x \cdot \tanh x$
6 $\text{csch}\, x$	$\longrightarrow$	$-\text{csch}\, x \cdot \coth x$

Ex: if $y = \tanh^3(e^{-x}) + \cosh x \cdot \text{sech}^5(x^2+4)$

find y' ?

Sol:-

$$y' = 3(\tanh(e^{-x}))^2 \cdot (\text{sech}^2(e^{-x})) \cdot (-e^{-x})$$

$$+ \cosh x \cdot 5(\text{sech}(x^2+4))^4 \cdot$$

$$(-\text{sech}(x^2+4) \cdot \tanh(x^2+4)) \cdot 2x$$

$$+ (\text{sech}(x^2+4))^5 \cdot \sinh x$$

EX :-

① $\sinh^{-1}(\sinh(-3)) = -3$

② $\sinh^{-1}(\sinh(4)) = 4$

③ $\cosh(\cosh^{-1}(4)) = 4$

④ $\cosh^{-1}(\cosh(-2)) = 2$

⑤ $\text{sech}^{-1}(\text{sech}(4)) = 4$

⑥ $\text{sech}(\text{sech}^{-1}(-4)) =$ undefind

ملاحظة :-

عند التعامل مع الانفيرس

لازم نخلي $(\cosh x , \text{sech} x)$

اقتران واحد لـ واحد

one to one

Relations :-

1. $\sinh^{-1}(x) = \ln\left(x + \sqrt{x^2+1}\right) , \quad x \in \mathbb{R}$

2. $\cosh^{-1}(x) = \ln\left(x + \sqrt{x^2-1}\right) , \quad x \geq 1$

3. $\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) , \quad -1 < x < 1$

$\therefore \quad \sinh(2) = \dfrac{e^{2} - e^{-2}}{2}$

But ! $\sinh^{-1}(2) = \ln(2 + \sqrt{5})$

Note :-

$\Rightarrow \operatorname{csch} x = \dfrac{1}{\sinh x}$

$\Rightarrow \coth x = \dfrac{1}{\tanh x}$

But ! $\Rightarrow \operatorname{csch}^{-1}(x) = \sinh^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow \coth^{-1}(x) = \tanh^{-1}\left(\frac{1}{x}\right)$

(Hyperbolic function) ول ل

حفظا :)

$f(x) \longrightarrow f'(x)$

1. $\sinh^{-1}(x) \longrightarrow \dfrac{1}{\sqrt{x^2+1}}$, $x \in \mathbb{R}$

2. $\cosh^{-1}(x) \longrightarrow \dfrac{1}{\sqrt{x^2-1}}$, $x > 1$

3. $\tanh^{-1}(x) \longrightarrow \dfrac{1}{1-x^2}$, $|x| < 1$

   $-1 < x < 1$

4. $\coth^{-1}(x) \longrightarrow \dfrac{1}{1-x^2}$ $|x| > 1$

   $x > 1 , x < -1$

5. $\operatorname{sech}^{-1}(x) \longrightarrow \dfrac{-1}{x\sqrt{1-x^2}}$, $0 < x < 1$

6. $\operatorname{csch}^{-1}(x) \longrightarrow \dfrac{-1}{|x|\sqrt{x^2+1}}$, $x \neq 0$

Ex: $\frac{d}{dx}\left(\sinh^{-1}(2x)\right) = ??$

Sol: $= \frac{1}{\sqrt{(2x)^2+1}} \cdot 2$

Ex:- $\frac{d}{dx} \tan^{-1}\left(\sin(5x)\right) = ??$

Sol: $\frac{1}{1-\sin^2(5x)} \cdot 5\cos(5x)$

$= \frac{5\cos(5x)}{\cos^2(5x)} = \frac{5}{\cos(5x)}$

#

chapter

"4"

Application of differentiation :-

* Sketch :-

[1] Domain $f(x)$

[2] x - intercept (let $y=0$)

y - intercept (let $x=0$)

[3] V. asym (ندرس النهاية عند اصفار المقام لـ $f(x)$)

[4] H. asym ($\lim\limits_{x \to \pm\infty} f(x)$)

[5] Symmetry

$f(-x) = f(x) \Rightarrow$ even function

$f(-x) = -f(x) \Rightarrow$ odd function

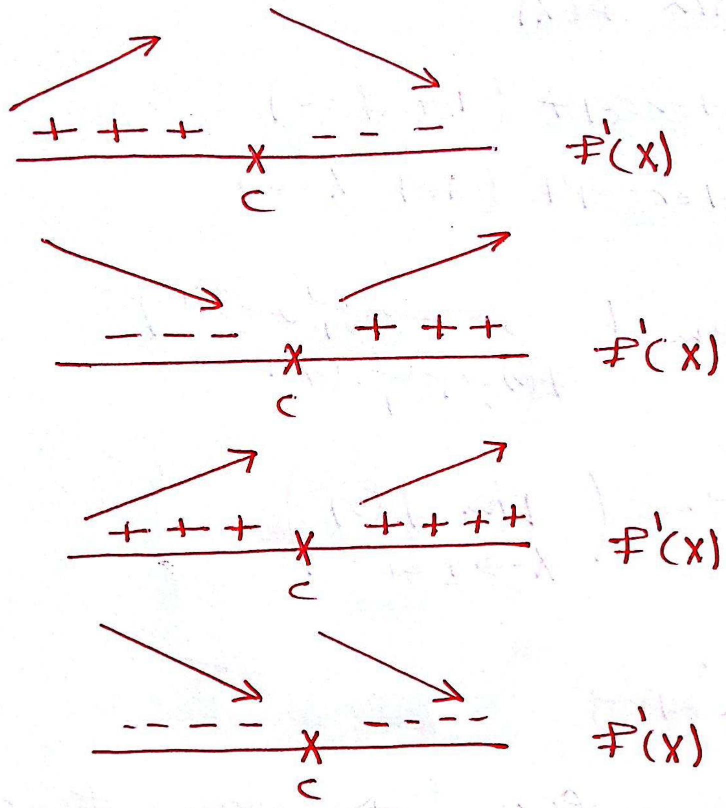
[6] Interval of increesing and decreesing of $f(x)$

$f'(c) = 0$ $\Downarrow$ اصفار البسط للمشتقة الأولى

$f'(c) = D.N.E$ $\Downarrow$ اصفار المقام للمشتقة التي داخل المجال + اطراف الفترة المغلقة

~~7 $f''(d) = 0$~~ , ~~$f''(d) = D.N.E$~~

⇒ first derevative test :-



⇒ second derivative test :-

$f''(c) > 0$ ⇒ at $x = c$ Local Min

$f''(c) < 0$ ⇒ at $x = c$ Local Max

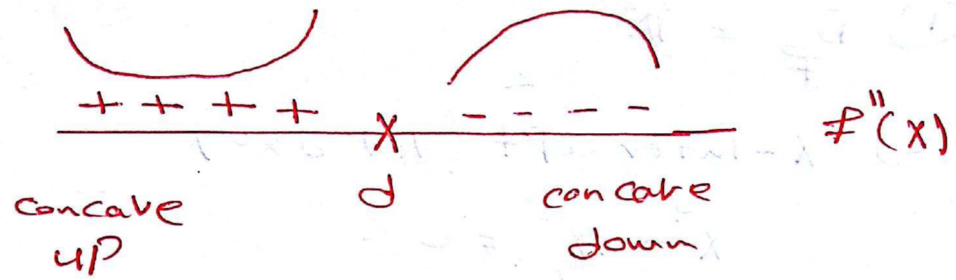
Ex :- if $f'(2) = 0$, $f''(2) = 10$

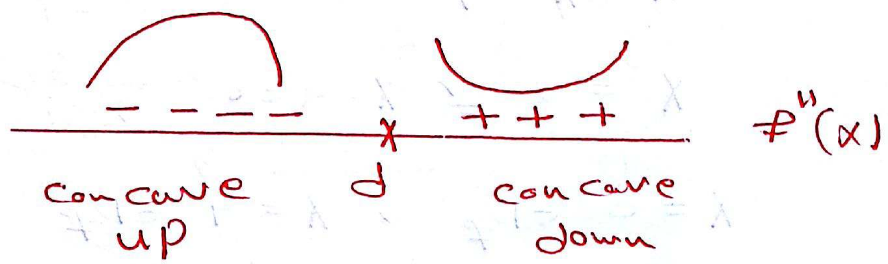
at $x = 2$ is ..Local min....

if $f'(2) = 0$, $f''(2) = -3$

at $x = 2$ is ..Local Max.

$\boxed{7}$ $f''(d) = 0$, $f''(d) = D.N.E$





at $x = d$ inflection Point.

<u>EX</u>:-

$f(x) = x^4 - 4x^3$ find:-

* سؤال شامل
عل اشي :)

① Domain $f(x)$

② x-intercept and y-intercept

③ V. asym (if exist).

④ H. asym (if exist).

⑤ symmetry (if exist).

⑥ The interval an which $f(x)$ is increesing and decreesing.

⑦ Local min and local max.

⑧ The interval an which $f(x)$ is concave up and concave down.

⑨ inflection Point

⑩ sketch graph

Solution:-

(1) $D_f = \mathbb{R}$

(2) X-intercept (let $y=0$)

$x^4 - 4x^3 = 0$

$x^3(x-4) = 0$

$x^3 = 0$ or $x-4=0$

$x = 0 \in D_f$, $x = 4 \in D_f$ $\Rightarrow (0,0), (4,0)$

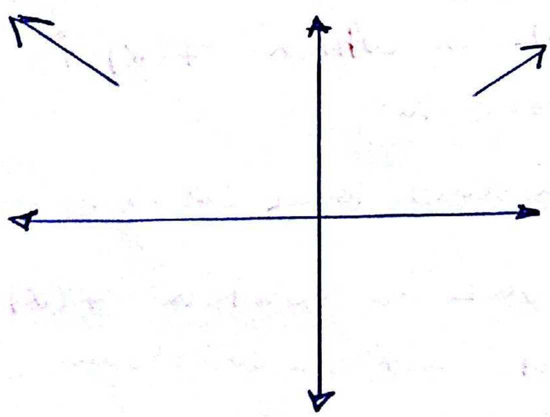
y-intercept (let $x=0$)

$y=0 \Rightarrow (0,0)$

(3) No V.A

(4) $\lim_{x \to \infty} x^4 - 4x^3 = +\infty$

$\lim_{x \to -\infty} x^4 - 4x^3 = +\infty$



⑤ $f(-x) = (-x)^4 - 4(-x)^3$

$= x^4 + 4x^3 \neq f(x) \longrightarrow$ Not even

$\neq -f(x) \longrightarrow$ Not odd

⑥ $f'(x) = 4x^3 - 12x^2$

$f'(x) = 0$

$4x^3 - 12x^2 = 0$

$4x^3(x-3) = 0$

$4x^3 = 0$ or $(x-3) = 0$

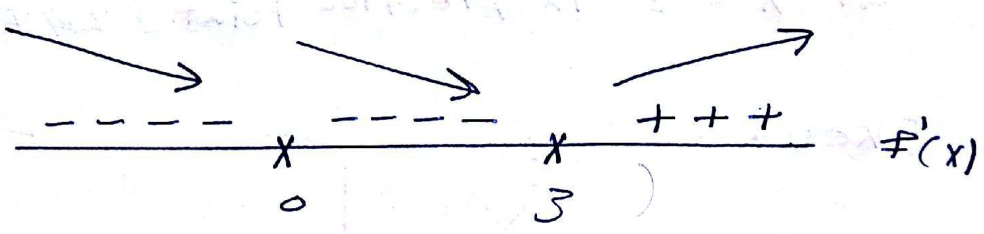
$x = 0 \in D_f$ $\quad$ $x = 3 \in D_f$

$f'(x) = D.N.E$

No

لا يوجد اصفار مقام
ولا فترة :D

at $x = 0$, $x = 3$ critical Points



f is decreesing $(-\infty, 0)$ and $(0, 3)$

f is increesing $(3, \infty)$

⑦ at $X = 3$ local min $(3, f(3))$

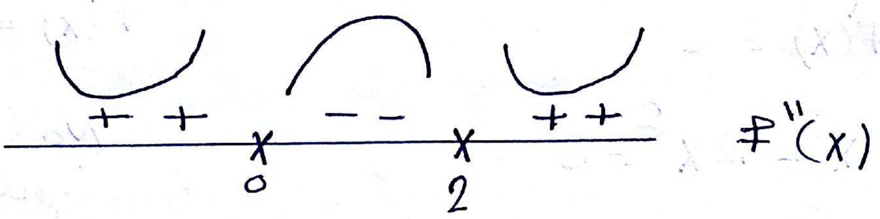
$= (3, -27)$

⑧ $f''(x) = 12x^2 - 24x$

$f''(x) = 0$

$12x(x-2) = 0$

$\boxed{x = 0}$ $\boxed{x = 2}$



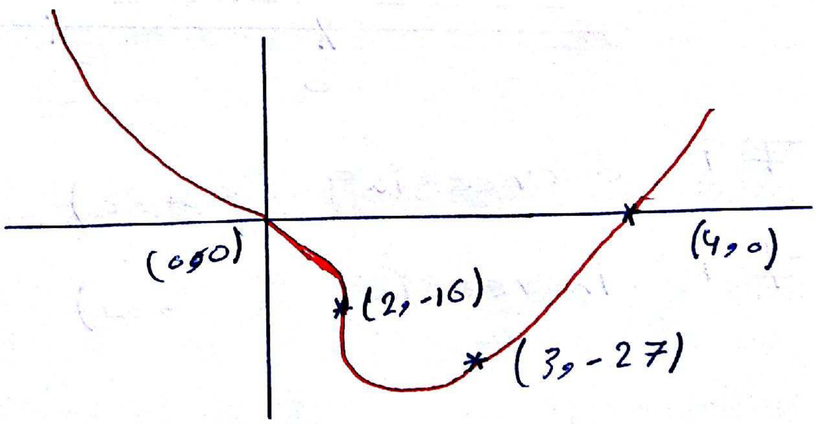
f is concave up on $(-\infty, 0)$ and $(2, \infty)$

f is concave down on $(0, 2)$.

⑨ at $x = 0$ inflection point $(0, f(0)) = (0,0)$

at $x = 2$ inflection point $(2, f(2)) = (2, -16)$

⑩ Sketch:-



** A function $f(x)$ is said to have:

[1] Absolute Max at c IF $f(c) \geq f(x)$, $\forall x$

[2] Absolute Min at c If $f(c) \leq f(x)$, $\forall x$

[3] local Max if $f(c) \geq f(x)$ in the neighborhod of c.

[4] Local Min if $f(c) \leq f(x)$, " " " " .

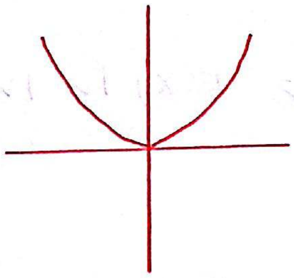
Thm 8- Extreme value
« القيم القصوى »

If $f(x)$ is cont on an interval $[a, b]$ then $f(x)$ has both absolute max and absolute min

* ⇒ A critical Number is the number that makes $f'(x) = 0$ or $f'(x) = D.N.E$

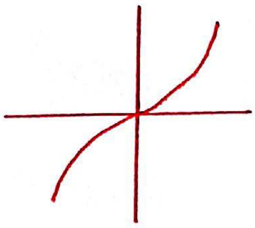
Ex:- Find Absolut and local values for each of the fotlwing :-

[1]



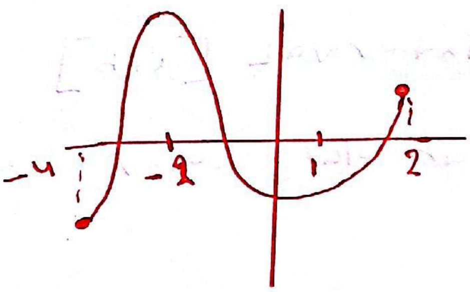
at $X = 0$ local min, and Absolute min at $X = 0$

[2]



No local, No Absolute

[3]



at $X = 1$, local min

at $X = -2$ local max

at $X = -4$ Absolut min

at $X = -2$ absolut max.

Note :- Methods to find absolute max and absolute min for cont. func. on closed intervals $[a, b]$

[1] find the critical number

[2] find the valule of critical number

[3] The largest value is absolute Max and the smallest value is absolute min

EX :- $f(x) = 4x^{\frac{3}{5}} - x^{\frac{8}{5}}$ find the critical number ?

Sol :- $f(x) = 4x^{\frac{3}{5}} - x^{\frac{8}{5}}$, $D = \mathbb{R}$

$$f'(x) = \frac{12}{5} x^{-2/5} - \frac{8}{5} x^{3/5}$$

$$= \frac{4}{5} x^{-2/5} (3 - 2x)$$

$$= \frac{4(3-2x)}{5x^{\frac{2}{5}}}$$

$f'(x) = 0$

$4(3 - 2x) = 0$

$x = \frac{3}{2} \in D_{\mathbb{R}}$

$f'(x) = D.N.E$

$5x^{2/3} = 0$

$x = 0 \in D_f$

$\Rightarrow$ critical $[0, 3/2]$

EX :- $f(x) = x^3 - 3x^2 + 1$, on $[-\frac{1}{2}, 4]$

find absolute max and min ?

Sol :- $D_f = [-\frac{1}{2}, 4]$

$f'(x) = 3x^2 - 6x$

$f'(x) = 0$

$f'(x) = D.N.E$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$\boxed{x = 0}$, $\boxed{x = 2}$

$\in D$ $\qquad \in D$

$\boxed{x = -\frac{1}{2}}$, $\boxed{x = 4}$ اطراف الفترة المغلقة

critical $\{0, 2, -\frac{1}{2}, 4\}$

$(0, f(0)) \Rightarrow (0, 1)$

$(2, f(2)) \Rightarrow (2, -3)$ → اصغر صورة

$(-\frac{1}{2}, f(-\frac{1}{2})) \Rightarrow (-\frac{1}{2}, \frac{1}{8})$

$(4, f(4)) \Rightarrow (4, 17)$ → اكبر صورة

at $x = 2$ Absolut min

at $x = 4$ Absolut max

⇒ $\boxed{H.W}$ ⇐ $f(x) = 6x^{4/3} - 3x^{1/3}$ on $[-1, 1]$

حلوه كالعسل :P

وهي الاجابات ⟸

at $x = \frac{1}{8}$ absolut min

at $x = -1$ absolut max

The Mean Value Thm. (M.V.T)

Roll's thm (مبرهنة حالة خاصة من M.V.T)

⇒ Let ① $f(x)$ is cont on $[a, b]$

② $f(x)$ is diff on (a, b)

③ $f(a) = f(b)$

∴ There is $c \in (a, b)$

EX:- $f(x) = \frac{1}{2}x^4 - x^2$ on $\left[-\frac{1}{2}, \frac{1}{2}\right]$ find c that statisfies Roll's thm ?

SoL:- ① f is cont on $\left[-\frac{1}{2}, \frac{1}{2}\right]$

② f is diff on $\left(-\frac{1}{2}, \frac{1}{2}\right)$

③ $f\left(-\frac{1}{2}\right) = f\left(\frac{1}{2}\right)$

⇒ by Roll's thm that $\exists\, c \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ يوجد معناها

$f'(c) = 0$

$2c^3 - 2c = 0$

$2c(c^2 - 1) = 0$

$2c(c-1)(c+1) = 0$

$\boxed{c = 0} \in \left(-\frac{1}{2}, \frac{1}{2}\right)$

* Mean value thm :-

① $f(x)$ is cont on $[a,b]$

② $f(x)$ is diff on (a,b)

$$\exists\ c \in (a,b) : f'(c) = \frac{f(b)-f(a)}{b-a}$$

<u>EX</u> :- $f(x) = x^3 - x$ on $[0,2]$, find c that statisfies Mean value thm. ?

<u>Sol</u> :- ① $f(x)$ is cont on $[0,2]$

② $f(x)$ is diff on $[0,2]$

$\exists\ c \in (0,2)$

$$f'(c) = \frac{f(2)-f(0)}{2-0}$$

$$3c^2 - 1 = \frac{6-0}{2-0}$$

$$3c^2 - 1 = 3$$

$c = \frac{2}{\sqrt{3}}$ ✓ or $c = -\frac{2}{\sqrt{3}} \notin (0,2)$ X

$$\boxed{c = \frac{2}{\sqrt{3}}}$$

Indeterminate forms :-

$\boxed{\frac{0}{0}\ ,\ \frac{\pm\infty}{\pm\infty}}$, $0\cdot\infty$, $\infty-\infty$, 1^{∞} , ∞^{0} , 0^{0}

↳ use l'HoPetal

$$\lim_{x\to a}\frac{f(x)}{g(x)} \Rightarrow \frac{f'(x)}{g'(x)}$$

$\boxed{0\cdot\infty}$

$$\lim_{x\to a} f(x)\cdot g(x) = 0\cdot\infty \ !!$$

$$\Rightarrow \lim \frac{f(x)}{\frac{1}{g(x)}} = \frac{0}{0} !! \text{ then use L'H}$$

<u>or</u> $$\lim \frac{g(x)}{\frac{1}{f(x)}} = \frac{\infty}{\infty} !! \text{ then use L'H}$$

$\boxed{\infty-\infty}$

- ضرب مرافق
- توحيد مقامات
- عامل مشترك

$\boxed{1^{\infty}\ ,\ \infty^{0}\ ,\ 0^{0}}$

$\Rightarrow$ ln (السؤال)

$0\cdot\infty$

$e^{(\text{الجواب})}$

* لم أتمكن من وضع أمثلة لضيق الوقت

لكن هي القواعد المهم :)

chapter

"5"

Integration

"التكامل"

$\int f(x)\,dx = F(x) + c$

(indefinit integral)

wher $F(x)$ an antiderivatve for $(f(x))$

⇒ Proparites :-

$\boxed{1}$ $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

$\boxed{2}$ $\int c\, f(x)\,dx = c \int f(x)\,dx$

ثابت

$\boxed{3}$ $\int f'(x)\,dx = f(x) + c$

ثابت التكامل

EX :- if $\int \sqrt{x^2+1}\,dx = f(x)$, find $f''(-1)$?

Sol :- $\cancel{\frac{d}{dx}}\int \sqrt{x^2+1}\,\cancel{dx} = \frac{d}{dx} f(x)$

$\sqrt{x^2+1} = f'(x)$

$\frac{2x}{2\sqrt{x^2+1}} = f''(x) \Rightarrow f''(-1) = \frac{-1}{\sqrt{2}}$

EX :- $\int g(x)\,dx = x^2 + 2x$, find : $g(x)$, $g(3)$?

Sol :-
$\Rightarrow$
$$\frac{d}{dx}\int g(x)\,dx = \frac{d}{dx}\left(x^2 + 2x\right)$$

$$g(x) = 2x + 2$$
$$g(3) = 2(3) + 2 = \boxed{8}$$

Integration Rules :-

$\boxed{1}$ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c$, $n \neq -1$

$\boxed{2}$ $\int x^{-1}\,dx = \ln|x| + c$

$\boxed{3}$ $\int e^x\,dx = e^x + c$

$\boxed{4}$ $\int b^x\,dx = \frac{b^x}{\ln b} + c$

$\boxed{5}$ $\int e^{ax+d}\,dx = \frac{e^{ax+d}}{a} + c$

$\boxed{6}$ $\int b^{ax+d}\,dx = \frac{b^{ax+d}}{a\ln b} + c$

Trigonometric Function :-

1. $\int \sin x \, dx = -\cos x + c$

2. $\int \cos x \, dx = \sin x + c$

3. $\int \sec^2 x \, dx = \tan x + c$

4. $\int \csc^2 x \, dx = -\cot x + c$

5. $\int \sec x \cdot \tan x \, dx = \sec x + c$

6. $\int \csc x \cdot \cot x \, dx = -\csc x + c$

Important Rule :-

$\Rightarrow$ اذا كان البسط مشتقة للمقام

$$\Rightarrow \int \frac{f'(x)}{f(x)} \, dx = \ln |f(x)| + c$$

---

7. $\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$

$$= -\ln |\cos x| = \ln |\cos x|^{-1}$$

$$= \ln |\sec x| + c$$

[8] $\int \cot x \, dx = \int \frac{\cos x}{\sin x} dx$

$= \ln|\sin x| + c$

Hyperbolic function:-

[1] $\int \sinh x \, dx = \cosh x + c$

[2] $\int \cosh x \, dx = \sinh x + c$

[3] $\int \operatorname{sech}^2 x \, dx = \tanh x + c$

[4] $\int \operatorname{csch}^2 x \, dx = -\coth x + c$

[5] $\int (\operatorname{sech} x \cdot \tanh x) dx = -\operatorname{sech} x + c$

[6] $\int (\operatorname{csch} x \cdot \coth x) dx = -\operatorname{csch} x + c$

Inveres Trigonomitric function:-

[1] $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c$

[2] $\int \frac{dx}{1+x^2} = \tan^{-1} x + c$

Ex :- ① $\int \frac{x^2 + 2\sqrt{x} + 5x}{x^2}\, dx = ??$

$$\Rightarrow \int \left(\frac{x^2}{x^2} + \frac{2x^{\frac{1}{2}}}{x^2} + \frac{5x}{x^2} \right) dx$$

$$= \int \left(1 + 2x^{-3/2} + 5x^{-1}\right) dx$$

$$= x - \frac{4}{\sqrt{x}} + 5 \ln|x| + c$$

② $\int 2^x\, dx = ??$

$$= \frac{2^x}{\ln 2} + c$$

③ $\int 2^x \cdot e^x\, dx = ??$

$$\Rightarrow \int (2e)^x\, dx = \frac{(2e)^x}{\ln(2e)} = \frac{(2e)^x}{\ln 2 + \ln e} + c$$

④ $\int 2^x \cdot 3^{2x+1}\, dx\ ??$

$$\Rightarrow \int 2^x (3^2)^x \cdot 3\, dx$$

$$= 3 \int (2(9))^x\, dx$$

$$= \frac{3(18)^x}{\ln(18)} + c$$

⑤ $\int \left(8x^2 + \frac{1+\cos^2 x}{\cos^2 x} + \frac{\sin x}{\cos^2 x} + \cos x \cdot \sec x\right) dx$

$\Rightarrow \int \left(8x^2 + \frac{1}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} + \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} + \cos x \cdot \frac{1}{\cos x}\right)$

$\Rightarrow \int \left(8x^2 + \sec^2 x + 1 + \sec x \cdot \tan x + 1\right) dx$

$= \frac{8x^3}{3} + \tan x + 2x + \sec x + c$

Integration by subsitution :-

>> التكامل بالتعويض <<

$\Rightarrow \int f(g(x)) \cdot g'(x)\, dx$

نفرض Let $u = g(x)$

$du = g'(x)\, dx$

$\Rightarrow \int f(u)\, du$

Ex:- $\int x\sqrt{x^2+4}\, dx$

Let: $u = x^2 + 4$

$du = 2x\, dx$

$dx = \frac{du}{2x}$

$\Rightarrow \int \cancel{x} \cdot u^{\frac{1}{2}} \cdot \frac{du}{2\cancel{x}}$

$= \frac{1}{2}\int u^{1/2}\, du = \frac{1}{\cancel{2}} \cdot u^{3/2} \cdot \frac{\cancel{2}}{3}$

$= \frac{1}{3}(x^2+4)^{3/2} + c$

EX :- $\int (3x+5)^{10}\, dx$

Let :- $u = 3x+5$

$du = 3\,dx$

$dx = \frac{du}{3}$

$\Rightarrow \int (u)^{10} \frac{du}{3}$

$= \frac{1}{3} \cdot \frac{u^{11}}{11} + c$

$= \frac{1}{33} (3x+5)^{11} + c$

EX :- $\int \left(\cos^3(2x) \cdot \sin(2x)\right) dx = ??$

Let :- $u = \cos(2x)$

$du = -2\sin(2x)\,dx$

$dx = \frac{du}{-2\sin(2x)}$

$\Rightarrow \int u^3 \cdot \cancel{\sin(2x)} \cdot \frac{du}{-2\,\cancel{\sin(2x)}}$

$= -\frac{1}{2} \int u^3\, du$

$= -\frac{1}{2} \cdot \frac{u^4}{4} + c = -\frac{1}{8} \left(\cos(2x)\right)^4 + c$

EX :- $\int \frac{\ln x}{x}\, dx = ??$

Let : $u = \ln x$

$du = \frac{1}{x}\, dx$

$dx = x\, du$

$\Rightarrow \int \frac{u}{\cancel{x}} \cdot \cancel{x}\, du$

$\Rightarrow \int u\, du$

$= \frac{u^2}{2} + c = \frac{(\ln x)^2}{2} + c$

Ex:- $\int (2x-1)(x+3)^{10}\, dx = ??$

Let:- $u = x+3$

$du = dx$

\# $\boxed{x = u-3}$

$$\Rightarrow \int (2x-1)(u)^{10}\, du$$

$$= \int \left(2(u-3)-1\right)(u)^{10}\, du$$

$$= \int 2u-7 \cdot u^{10}\, du$$

$$= \int 2u^{11} - 7u^{10}\, du$$

$$= \frac{2u^{12}}{12} - \frac{7u^{11}}{11} + c$$

$$= \frac{2(x+3)^{12}}{12} - \frac{7(x+3)^{11}}{11} + c$$

Ex:- $\int \frac{dx}{\sqrt{4-x^2}} = ??$

$$= \int \frac{dx}{\sqrt{4\left(1-\frac{x^2}{4}\right)}} = \frac{1}{2}\int \frac{dx}{\sqrt{1-\left(\frac{x}{2}\right)^2}}$$

Let: $u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$dx = 2du$

$$\Rightarrow \cancel{\tfrac{1}{2}} \int \frac{\cancel{2}du}{\sqrt{1-(u)^2}}$$

$$\Rightarrow \int \frac{1}{\sqrt{1-(u)^2}}\, du$$

$$= \sin^{-1}(u) + c$$

$$= \sin^{-1}\left(\frac{x}{2}\right) + c$$

Ex:- $\int \frac{dx}{5+3x^2} = ??$

$$\Rightarrow \int \frac{dx}{5\left(1+\frac{3x^2}{5}\right)} = \frac{1}{5}\int \frac{dx}{1+\left(\frac{\sqrt{3}\,x}{\sqrt{5}}\right)^2}$$

Let :- $u = \frac{\sqrt{3}\,x}{\sqrt{5}}$

$du = \frac{\sqrt{3}}{\sqrt{5}}\,dx$

$dx = \frac{\sqrt{5}}{\sqrt{3}}\,du$

$$= \frac{1}{5}\int \frac{\frac{\sqrt{5}}{\sqrt{3}}\,du}{1+(u)^2}$$

$$= \frac{1}{\sqrt{15}}\tan^{-1}(u) + c$$

$$= \frac{1}{\sqrt{15}}\tan^{-1}\left(\frac{\sqrt{3}\,x}{\sqrt{5}}\right) + c$$

Ex:- $\int \frac{x}{5+3x^2}\,dx = ??$

Let :- $u = 5+3x^2$

$du = 6x\,dx$

$dx = \frac{du}{6x}$

$$\Rightarrow \int \frac{\cancel{x}}{u}\cdot\frac{du}{6\cancel{x}}$$

$$= \frac{1}{6}\int u^{-1}\,du$$

$$= \frac{1}{6}\ln|u| + c$$

$$= \frac{1}{6}\ln|5+3x^2| + c$$

Rule :-

$$\frac{d}{dx}\int_{h(x)}^{g(x)} f(t)\,dt$$

$$= f(g(x))\cdot g'(x) - f(h(x))\cdot h'(x)$$

EX :- $$\frac{d}{dx}\int_{\cos x}^{\sinh x}\sqrt{1+t^2}\,dt \quad ??$$

$$\Rightarrow \sqrt{1+(\sinh)^2}\cdot \cosh x - \sqrt{1+(\cos x)^2}\cdot(-\sin x)$$

Rule :-

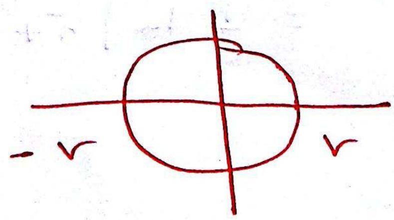
$$\int_{-r}^{r}\sqrt{r^2-x^2}\,dx = \frac{1}{2}(\pi r^2)$$

($\frac{1}{2}$ مساحة دائرة)

$$\int_{0}^{r}\sqrt{r^2-x^2}\,dx = \frac{1}{4}(\pi r^2)$$

($\frac{1}{4}$ مساحة دائرة)

Note :- $$x^2+y^2=r^2$$



$$y=\sqrt{r^2-x^2}$$

EX:- [1] $\int_{-2}^{2} \sqrt{4-x^2}\, dx = ??$

$r^2 = 4$

$r = 2$ $\Rightarrow \frac{1}{2}(\pi(2)^2) = 2\pi$

[2] $\int_{0}^{3} \sqrt{9-x^2}\, dx = ??$

$r^2 = 9$

$r = 3$ $\Rightarrow \frac{1}{4}(\pi(3)^2) = \frac{9}{4}\pi$

$\Longrightarrow$ How to find the area by using the integration?

$f(x) \geq 0$

$\int_{a}^{b} f(x)\, dx \geq 0$ (فوق محور السينات)

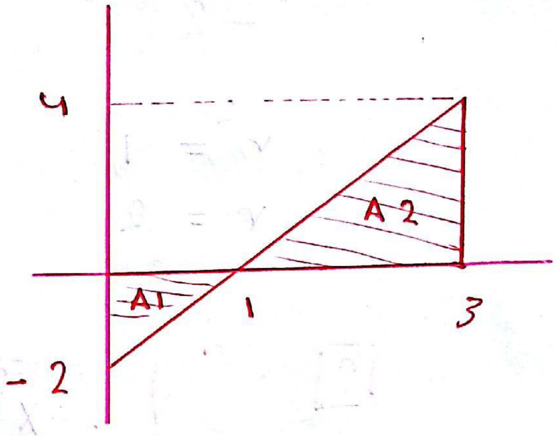
$f(x) \leq 0$

$\ominus \int_{a}^{c} f(x)\, dx \leq 0$ (تحت محور السينات)

EX :-

find :-

1) $\int_0^3 f(x)\,dx$



$= \int_0^1 f(x)\,dx + \int_1^3 f(x)\,dx$

$= -1 + 4 = \boxed{3}$

$A_1 = \frac{1}{2}(1)(2) = \boxed{1}$

$A_2 = \frac{1}{2}(2)(4) = \boxed{4}$

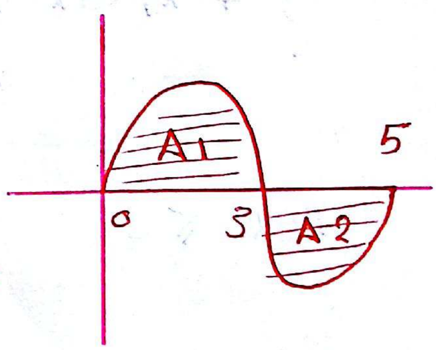
2) Area $f(x)$ on $[0, 3]$

$\Rightarrow$ Area $= A_1 + A_2$

$= 1 + 4 = \boxed{5}$

EX :- If $\int_0^3 f(x)\,dx = 6$,

$\int_0^5 f(x)\,dx = 4$



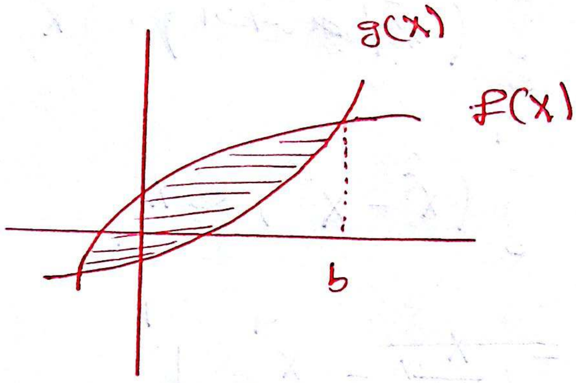
find area $f(x)$ on $[0, 5]$??

$\Rightarrow \int_0^5 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx$

$4 = 6 + \int_3^5 f(x)\,dx$, $\int_3^5 f(x)\,dx = -2$

Area $f(x) = A_1 + A_2 = |6| + |-2| = 8$

Note :-



Area between $f(x)$ and $g(x)$!

$$\# \quad \text{Area} = \int_a^b \left(f(x) - g(x) \right) dx$$

!! $\boxed{\boxed{\text{Note}}}$ $\text{Area} = \int_a^b \boxed{\text{الأكبر}} \xRightarrow{\text{ناقص}} \boxed{\text{الأصغر}}$

Ex :- find area between $y_1 = x^2$ and $y_2 = x^3$

$y_1 = y_2 \Rightarrow$ منها نطلع حدود التكامل

$x^2 = x^3$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$ $\quad \boxed{x=0} \quad \boxed{x=1}$

⟸ طيب السؤال هنا ؟؟ مين الأكبر ومين الأصغر ؟؟

⟸ هذا حدود التكامل ع خط أعداد زي هيك

واختار رقم بينهم ، وعوض الرقم في

الاقترانين الجواب الأكبر ⟸ يكون هو الأكبر

$0 \quad \frac{1}{2} \quad 1$

$\frac{1}{4} > \frac{1}{8}$ $\qquad y_1 = \left(\frac{1}{2}\right)^2 = \boxed{\frac{1}{4}}$

$\therefore y_1 \Rightarrow$ الأكبر $\qquad y_2 = \left(\frac{1}{2}\right)^3 = \boxed{\frac{1}{8}}$

$$\text{Area} = \int_0^1 \left(\text{الأصغر} \;-\; \text{الأكبر} \right) dx$$

$$\Rightarrow \int_0^1 (x^2 - x^3)\, dx$$

$$= \frac{x^3}{3} - \frac{x^4}{4} \Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}}$$

» أسئلة سنوات متنوعة «

$\boxed{1}$ $\displaystyle\int \frac{\sin(2x)}{\cos(x)} = ??$

$$= \int \frac{2\sin x \cdot \cancel{\cos x}}{\cancel{\cos x}}\, dx = \int 2\sin x\, dx$$

$$= -2\cos x + c$$

$\boxed{2}$ $\displaystyle\int_{-10}^{10} \frac{2e^x}{\sinh x + \cosh x}\, dx = ??$

$$= \int_{-10}^{10} \frac{2e^x}{e^x}\, dx = \int_{-10}^{10} 2\, dx$$

$$= 2x \Big]_{-10}^{10} = 2(10) - 2(-10)$$

$$= \boxed{40}$$

$$= \int e^{\sqrt{(t+1)^2}} \, dt$$

$$= \int e^{t+1} \, dt = e^{t+1} + c$$

ike

$\boxed{4}$ $\displaystyle\int \frac{\sin x}{2 - \sin^2 x} \, dx = ??$

$$2 - \sin^2 x \longrightarrow 1 + \underline{1 - \sin^2 x}$$

$$\int \frac{\sin x}{1 + \cos^2 x} \, dx$$

Let $u = \cos x$

$du = - \sin x \, dx$

$$\Rightarrow - \int \frac{du}{1+u^2} = - \tan^{-1} u$$

$$= - \tan^{-1} (\cos x)$$

$\boxed{5}$ $\displaystyle\int \sin^2 x \, dx = ??$

$$= \int \frac{1}{2} (1 - \cos 2x) \, dx$$

$$= \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right] + c$$